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ABSTRACT

"Primary Mathematics" is a magazine for primary grades teachers. It contains activities which can be used, or adapted for use, with children over a wide ability range. It also contains articles on the teaching and learning of mathematics. The material in this book was selected from 18 issues of "Primary Mathematics" published during the period from 1980-85. Questions from the annual Primary Mathematics Competition have been used to fill up gaps in the text. Topics include numbers, pattern recognition, arithmetic, line symmetry, magic squares, map skills, multiplication, problem solving, hexagons, division, binary numbers, field trips, remainders, tessellations, decision making, tangrams, place value, factors, and shapes. (KR)

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A Selection of Materials from Primary Mathematics 1980-85

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MATHS MATTERS

A Selection of Materials from

Primary Mathematics 1980 ~ 85

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Acknowledgments

Mike Palmer, Primary Mathematics Curriculum Officer, 1986

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Maths Resource Centre, Hobart Tasmania

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PREFACE

'Primary Mathematics' is a magazine for primary teachers. It contains mathematical activities which can be used, or adapted for use, with children over a wide ability range. It also contains articles on the teaching and learning of mathematics. Many important issues are raised and these would be most suitable as a basis for discussion at pre-service and in-service sessions.

The material in this book has been selected from the 18 issues of 'Primary Mathematics' which I wrote and distributed to Tasmanian schools during the period 1980-85. Questions from the annual Primary Mathematics Competition have been used to fill up gaps in the text.

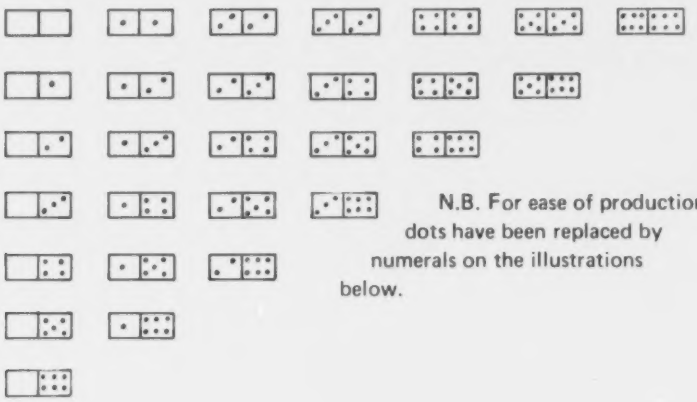
Pages 71 and 108 are from the book 'Notes on Mathematics for Children' by the Association of Teachers of Mathematics, U.K., 1977, and are reproduced here with the permission of the publishers, Cambridge University Press.

I would like to thank Kathy Godfrey for doing all the typing and diagrams and Pat Jeffery for suggesting the title 'Maths Matters'.

Michael Palmer
Curriculum Officer Mathematics

ACTIVITIES USING DOMINOES K → 6

The activities suggested below use the most common set of dominoes available in toyshops: the "double-six" set.



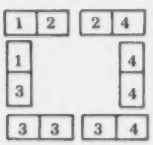
N.B. For ease of production dots have been replaced by numerals on the illustrations below.

Each number has its own distinct pattern of dots and therefore simple matching games can be played even though the children may not know the number names. All that is needed is one-to-one correspondence and not the ability to count; although the dominoes would help in this regard.

1. Make a domino train by matching the dot patterns.

Can you make a train using all the dominoes?

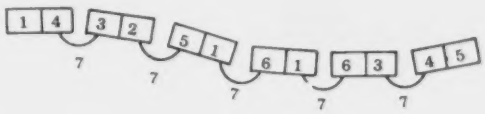
2. Make a loop.



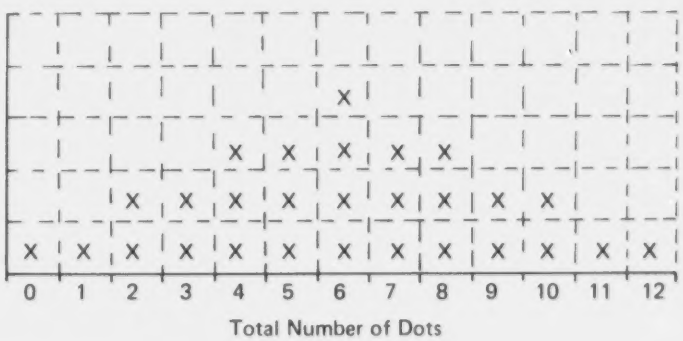
Can you make a loop using all the dominoes?

3. a) How many dominoes have 3 dots (this pattern ⋰)?
   b) How many dominoes have 6 dots (this pattern ⁝)?
   c) How many dominoes have no dots?

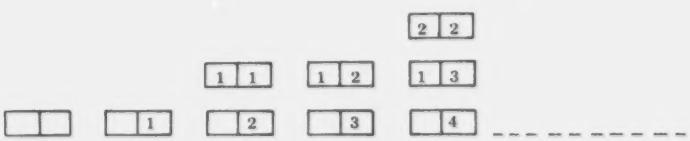
4. Make a domino train, but instead of matching numbers, match so as to give a particular total - say 7.



5. Place the dominoes face downwards on the floor.
   Pick up any domino and count the total number of dots.
   Record the answer on a graph.
   Keep this domino, pick up another and record the total.



The domino graph can be built using the actual dominoes.



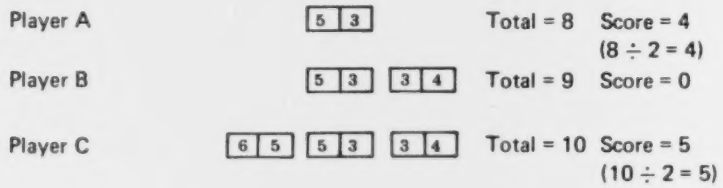
6. Domino Games: 2-5 players

The dominoes are placed face down and each player selects five.

The dominoes are matched in the normal way and each player tries to play so that the sum of the numbers at the ends of the train is a multiple of 2.

The players score one point for each 2 they make in this way.

E.g.

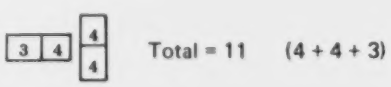


If a player "can't go" he picks up a domino from those not shared out at the beginning and the play moves on to the next person.

The winner is the player with the highest score when all dominoes have been played.

The game can be varied by altering the scoring multiple to 3 instead of 2, or to 5. In this form it can be played quite successfully by 8 year olds and at the same time reinforcing many of the basic number skills.

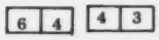
A more advanced version allows "doubles" to be placed at right angles to the line thus increasing the total.

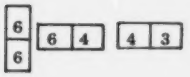


The scoring can also be changed to multiples of 3 or 5 or both.

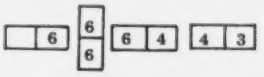
E.g.

Player A Total = 10 Score = 2 (10 ÷ 5 = 2)

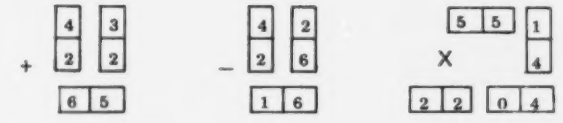
Player B  Total = 9 Score = 3 (9 ÷ 3 = 3)

Player C  Total = 15 Score = 8

15 ÷ 5 = 3
15 ÷ 3 = 5
3 + 5 = 8

Player D  Total = 3 Score = 1 (3 ÷ 3 = 1)

7. Dominoes can be placed so as to make up very simple problems.



43 + 22 = 65 42 – 26 = 16 551 X 4 = 2204

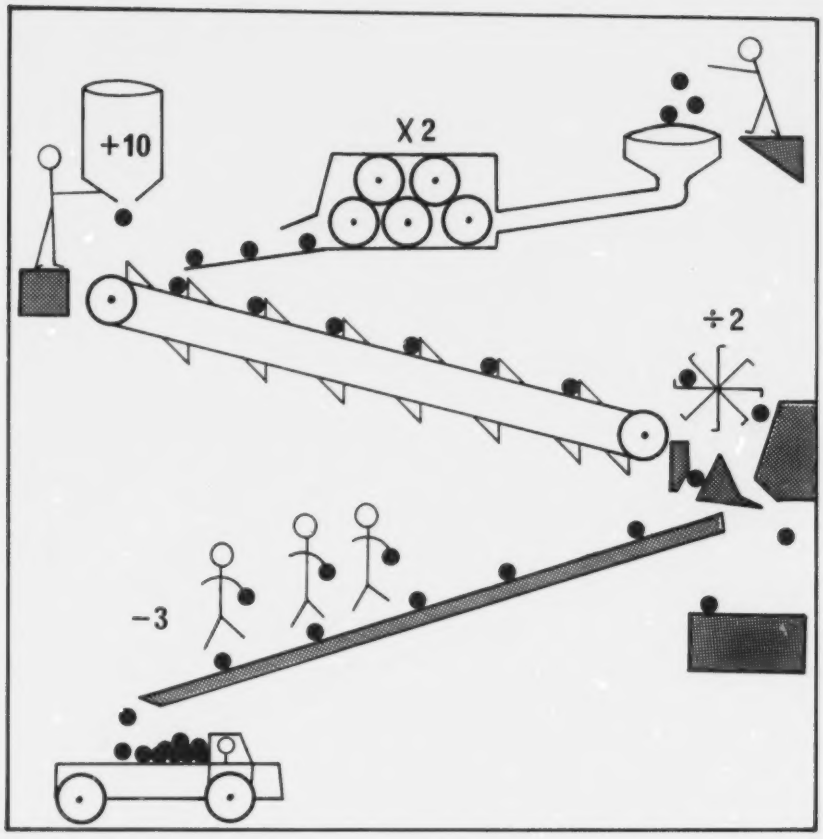
Make up a problem which uses the largest number of dominoes.

8. Domino Puzzle

A complete set of dominoes fell out of the box and landed on the floor. By a truly remarkable accident they fell in a pattern exactly like the one opposite. Can you locate the individual dominoes?

5	1	4	6	0	3	3	5
6	5	4	6	2	2	4	0
4	5	4	5	0	0	2	5
6	2	1	3	3	6	3	0
4	2	3	5	0	1	6	6
0	1	4	1	4	1	5	6
2	1	3	2	0	3	2	1

NUMBER MACHINES



The diagram shows a number machine.

a) If you put 5 into the machine, what number will end up in the delivery truck.

b) Try some more numbers. Keep a record of your results. Describe what the machine does to numbers.

c) If you want to end up with 16 in the delivery truck, what number must you put into the machine?

d) Make up your own number machine and show it to your teacher.

* * *

BUILDING HOUSES IN MATHSLAND

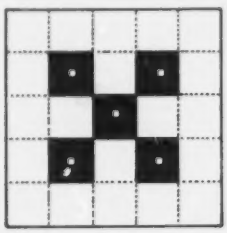
Materials required per child: worksheets and 5 unifix cubes (initially).

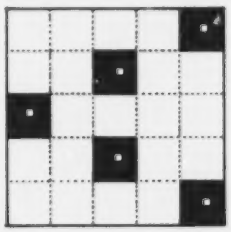
The large square on the worksheet is the plan of a building site. It is divided into 25 smaller squares and has a perimeter of 20 units. The Unifix cubes represent rooms. Each room has four walls, no floor and a ceiling or roof, with a hole in it.

In Mathsland all building sites are exactly the same shape and size. When houses are built the rooms are placed exactly on the small squares of the site.

Use the 5 cubes to make a model of a 'one-storey' house.

If this is the only instruction given, some children will make houses similar to those below.





They can then be asked questions such as:

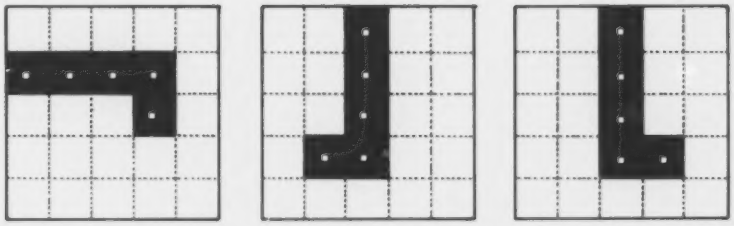
Which of the rooms is your bedroom?
Which room is the kitchen?
How would you get from the kitchen to your bedroom?
Can you walk through a corner?

This should lead to the formulation of a rule that rooms must be joined exactly along a common wall.

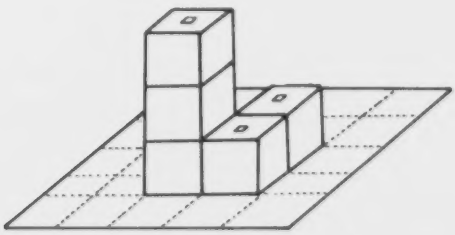
How many different 'one-storey' houses is it possible to build?

Are these the same?

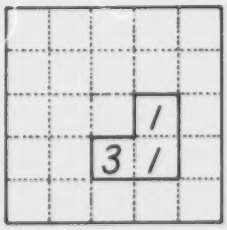
What about these?



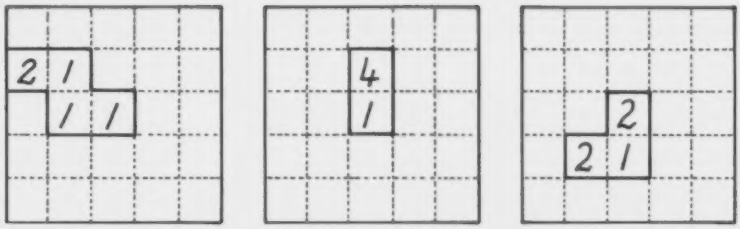
Make a building like this:
What would it look like from
an aeroplane flying overhead?



Draw the outline of the building.
Write in each square the number
of rooms that stand on that
square.

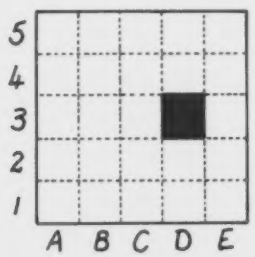


Make these buildings:



How could you describe the site plan of your building to another person on the telephone?

Label the rows and columns of your building site as shown. The position of a room placed on the shaded square could be described: D3.
If there were 4 rooms on this square it could be described as (D3, 4).
(E2, 3) means on the position E2, the building has 3 rooms.



Build the house given by: (B3, 1); (C3, 1); (C2, 2); (D2, 1).

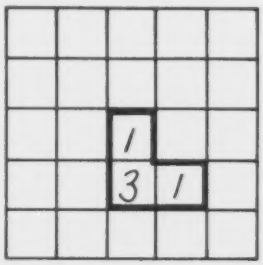
The cost of building a house is determined by:

a) the number of rooms on each level (storey);
b) the number of outside wall sections;
c) the number of roof sections.

Because of low-flying aircraft no house in Mathsland can be more than 5 stories high. The higher the building the greater the cost. It costs $1-00 to place a room on the ground floor, $2-00 for each room on the 2nd level, $3-00 for the 3rd level etc.
Each outside wall section costs $1-00.
Each roof section costs $2-00.

Before planning permission is granted a form giving the site plan, details of cost and description of the house must be completed.

SITE PLAN



COST

POSITION	NUMBER OF ROOMS	COST PER ROOM $	AMOUNT
Level 1	3	1	3
Level 2	1	2	2
Level 3	1	3	3
Level 4		4	
Level 5		5	
		TOTAL	8

NUMBER OF OUTSIDE WALL SECTIONS	UNIT COST $	TOTAL
16	1	16

NUMBER OF ROOF SECTIONS	UNIT COST $	TOTAL
3	2	6

TOTAL COST OF BUILDING

8
16
6
$ 30

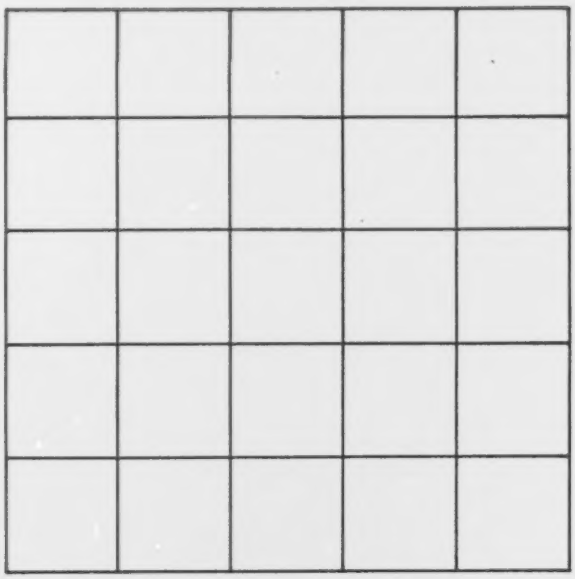
DESCRIPTION

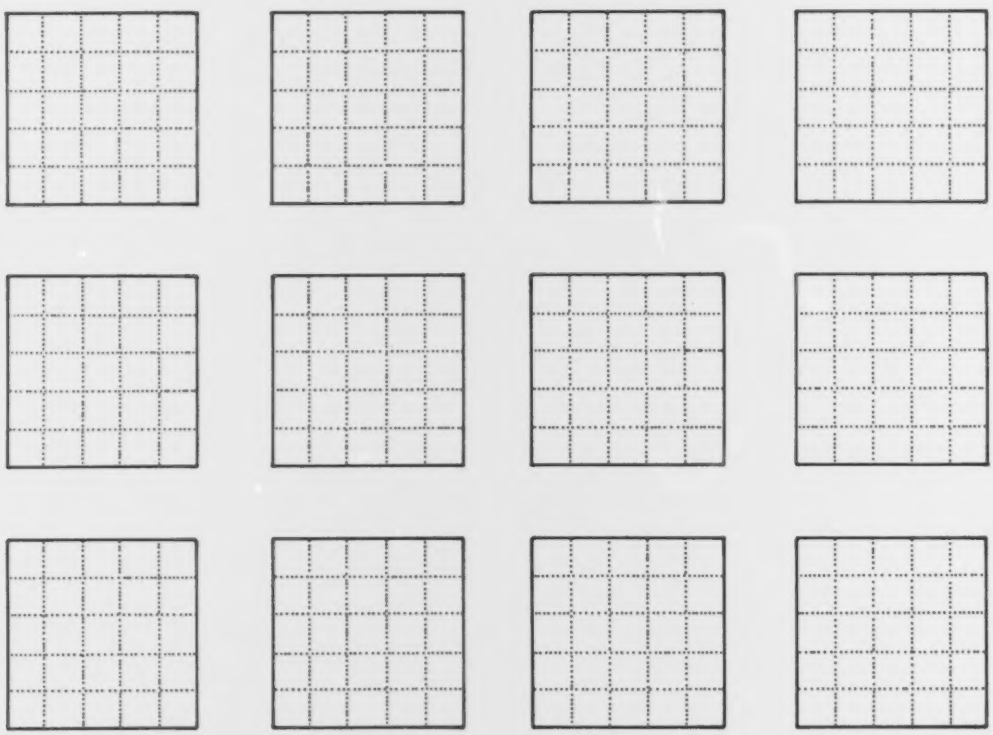
My house is blue and has a garden all the way round. It cost $30 to build. It has 3 chimneys.

Calculate the maximum and minimum cost of a 5 roomed house.

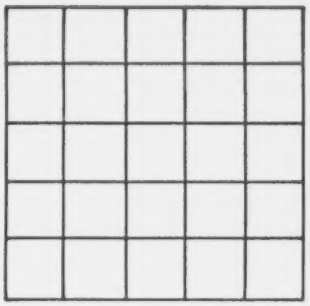
Design a house which costs exactly $50-00 (no restriction on the number of rooms, but no more than 5 stories high).

* * *





SITE PLAN



COST

POSITION	*NUMBER OF ROOMS*	*COST PER ROOM $*	*AMOUNT*
LEVEL 1		*1*	
LEVEL 2		*2*	
LEVEL 3		*3*	
LEVEL 4		*4*	
LEVEL 5		*5*	
		TOTAL	

NUMBER OF OUTSIDE WALL SECTIONS	*UNIT COST $*	*TOTAL*
	1	

NUMBER OF ROOF SECTIONS	*UNIT COST $*	*TOTAL*
	2	

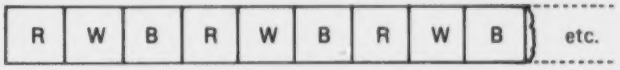
TOTAL COST OF BUILDING

$

DESCRIPTION

UNIFIX PATTERNS

Take a number of red, white and blue cubes and put them together to form the pattern:

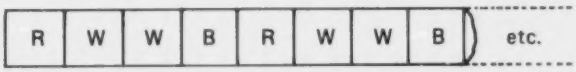


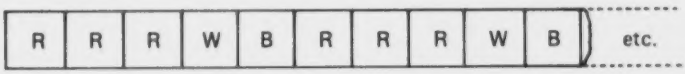
a) In what position are the blue cubes?

b) What colour would the 23rd, 36th, 40th and 51st cube be?

c) If the pattern is made from 55 cubes how many are red?

Answer the above questions for each of the patterns below:





Take 1 red, 1 white and 2 blue cubes and build a tower.

How many different towers can you build with these 4 cubes?

* * *

$5 = 3 \times 4 - 5 - 2$

2, 3, 4, 5

$56 = 34 - 25$

How many of the numbers: 1, 2, 3, 4, ... 100 can you make using equations that contain only the four digits 2, 3, 4 and 5?

RULES:

a) You must use 2, 3, 4, 5 each time but the order in which you write them down is unimportant.

b) You may use +, -, x, ÷ and brackets when necessary.

c) You may form two-digit or three-digit numbers.
E.g. 23, 52, 432 ...

d) If you are unable to obtain a number then you may extend rule (b) to include: powers, square roots, decimals and factorials.

$55 = 54 + 3 - 2$

$90 = 3 \times 5 \times (4 + 2)$

This is a good 'whole class' activity. It can create a great deal of excitement and provide an opportunity to reinforce and introduce many aspects of number work.

I have found that in working with children on this problem it is best to exhaust all the possibilities using only rules (a) and (b). Initially it is possible for all children to succeed; for just by trial and error an answer may be obtained. It soon becomes necessary to explain order of operations and the correct use of brackets, viz:

- If a number statement contains only + and - signs; work from left to right.
  $3 + 5 - 4 - 2 = 2$, $5 - 2 + 4 - 3 = 4$
- If a number statement contains only x and ÷ signs; work from left to right.
  $3 \times 4 \times 5 \div 2 = 30$, $4 \times 3 \div 2 \times 5 = 30$

- If the statement contains a mixture of the above then x and ÷ are worked before + and -.

  $4 \times 3 + 2 \times 5 = 22$, $5 + 3 - 4 \div 2 = 6$

- Brackets are always worked first. They are inserted in an expression when the rules above would give an answer different from that which is required:

without brackets	$3 \times 5 \times 4 + 2 = 62$
with brackets	$3 \times 5 \times (4 + 2) = 90$
without brackets	$2 \times 43 - 5 = 81$
with brackets	$2 \times (43 - 5) = 76$

Another feature of the problem is that a variety of different solutions can be obtained for the same number.
E.g. $24 = 34 - 2 \times 5$, $4 \times (5 + 3 - 2)$, $2 \times (3 + 4 + 5)$, ...

Rule (c) is introduced when solutions start to dry up. It is a good idea to get the children to make a list of all the 2 digit numbers that can be made.

23	32	42	52
24	34	43	53
25	35	45	54

These can then be systematically combined with the remaining numbers.

E.g. $54 + 32 = 86$, $42 - 35 = 7$, $45 + 23 = 68$
$23 + 4 - 5 = 22$, $25 - 3 \times 4 = 13$, $52 \div 4 - 3 = 10$

88 of the 100 numbers can be obtained using rules (a), (b) and (c). In order to get solutions for the remaining 12 numbers, more advanced mathematics may have to be introduced. The opportunity exists for work on indices, square roots, decimals and even factorials:

Indices: $2^3 = 8$, $2^4 = 16$, $2^5 = 32$
$3^2 = 9$, $3^4 = 81$, $3^5 = 243$
$4^2 = 16$, $4^3 = 64$, $4^5 = 1024$
$5^2 = 25$, $5^3 = 125$, $5^4 = 625$

Square Roots: $\sqrt{4} = 2, \quad \sqrt{25} = 5$

Decimals: $\cdot 2 = \frac{2}{10} = \frac{1}{5}, \quad \cdot 3 = \frac{3}{10}, \quad \cdot 4 = \frac{4}{10} = \frac{2}{5}, \quad \cdot 5 = \frac{5}{10} = \frac{1}{2}$

Factorials: The factorial of a number is the product of that number and all the consecutive numbers from the number down to 1. The mathematical symbol to denote this is an exclamation mark.

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ (read as 'factorial five')
$4! = 4 \times 3 \times 2 \times 1 = 24$
$3! = 3 \times 2 \times 1 = 6$
$2! = 2 \times 1 = 2$

Factorials are used in higher mathematics in connection with permutations and combinations.

Armed with this array of mathematics the possibilities are endless and it is not too difficult to find the remaining numbers.

$3^4 - 25 = 56$, $3 \times 2^5 - 4 = 92$ $2 \times 35 \div \sqrt{4} = 35$
$3^4 + 2 \times \cdot 5 = 82$, $3 \times 2^5 - \sqrt{4} = 94$ $5! + 3 - 24 = 99$

A regular feature of 'Primary Mathematics' over the years has been to use the digits of the current year to make equations for the numbers 1 to 100.

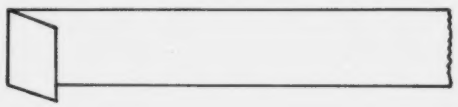
CHALLENGE: Solve the problem using the digits - 1, 9, 8, 6 .

* * *

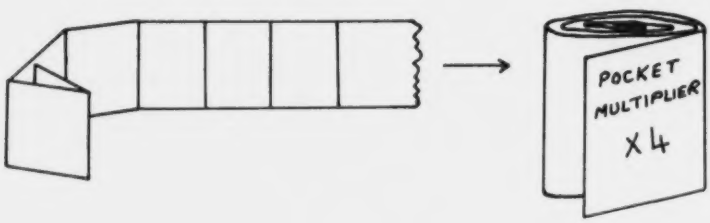
INSTRUCTIONS FOR MAKING POCKET MULTIPLIERS

Take a strip of paper approximately 60 cm long and 6 cm wide.

Fold over a rectangle approximately 4 cm wide at the left hand end of the strip.



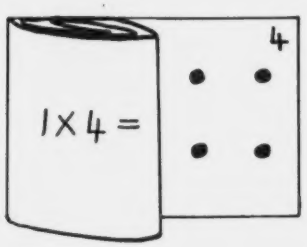
Continue to fold the strip over on itself for its entire length.
Cut off any surplus paper so as to form a nice compact rectangle.



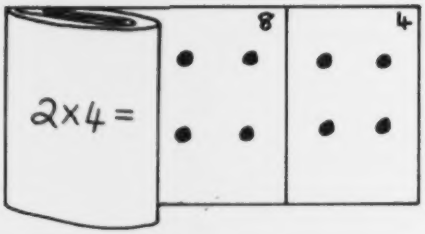
Choose the multiplier that you wish the child to study, e.g. 4, and write 'Pocket Multiplier X 4' on the front rectangle as shown in the diagram above.

Turn back the front rectangle and on the left side write 1 x 4 = and on the right make a pattern of 4 dots.

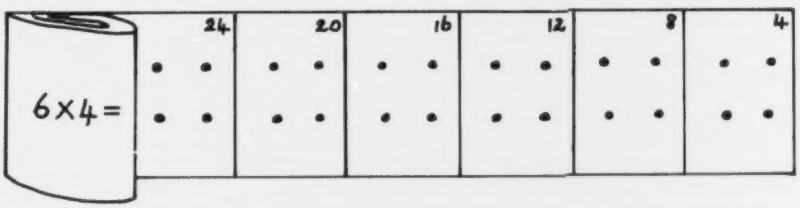
Write the number 4 in the top right hand corner.



Open the strip of paper one more rectangle to the left and this time write 2 x 4 = on the right and make another pattern of 4 dots. Write the number 8 in the top right corner.



Continue to open up the strip of paper and each time add one more multiple of 4.

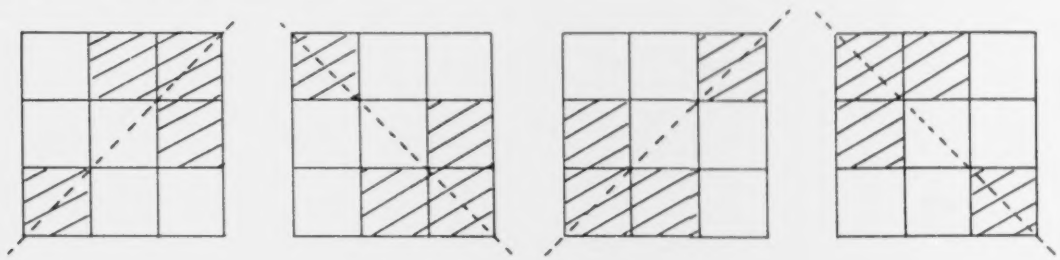


* * *

Take 4 black and 5 white square tiles.
Arrange them in a 3 x 3 square so that the resulting pattern of black and white tiles has line symmetry.

How many <u>different</u> patterns can you find?
Record your patterns on the squared paper provided and draw in the axis of symmetry with a dotted line.

Reflections and rotations do not count as new patterns.
E.g. These are the same:



*

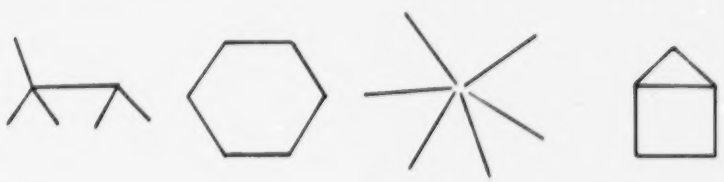
6

Choose a number - any number.
Think about it.
Draw it.
Count it.
Represent it.
Find it.
Cut it out.
Talk about it.
Write about it.

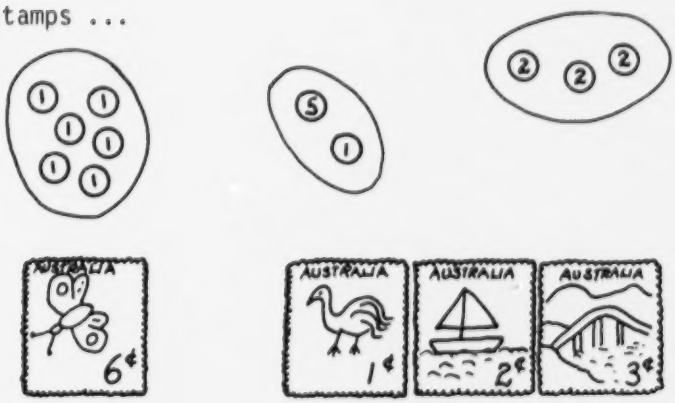
The story of 6

Marbles, centicubes, gum nuts, bottle tops, cars, children, dots ...

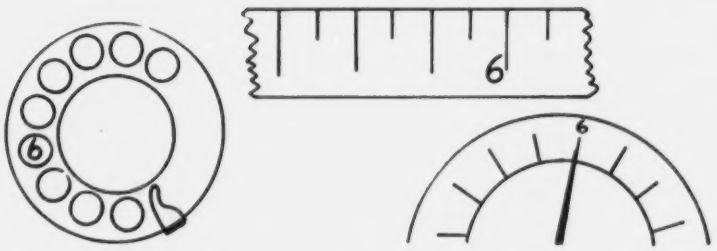
Pencils, matchsticks, straws, lines ...



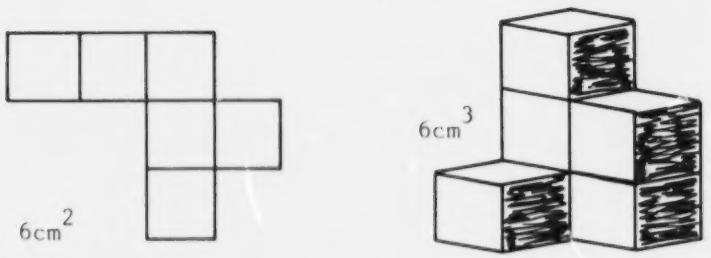
Coins and stamps ...



Dials and scales ...

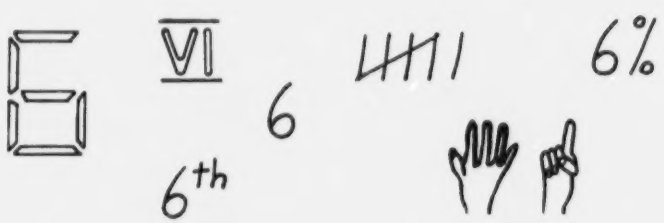


Squares and cubes ...



Measurement ...

Signs and symbols ...

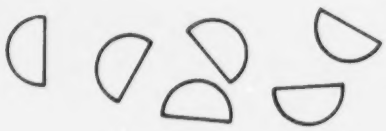


Arithmetic ...

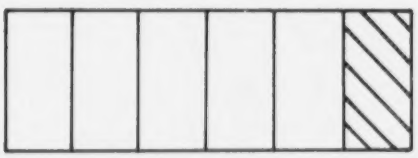
1 + 1 + 1 + 1 + 1 + 1, 5 + 1, 4 + 2, 1 + 2 + 3, 2 x 3,

$\frac{1}{2}$ of 12, 13 - 7

436.5 463.5 643.5 435.6



6 x $\frac{1}{2}$



$\frac{1}{6}$ sixth

Words and figures ...

My birthday is on the sixth.
Allan Border scored five 6's.
I came sixth in the race.
The butterfly is on page 6.
All ants have six legs.
My book cost $6.
There are 6 days until the end of term.
My mother asked me to buy a half dozen eggs.
'Perfect Match' is on at 6 o'clock.
I get off at Bus Stop 6.

TRY THIS ACTIVITY WITH YOUR CLASS TOMORROW.

TAKE FOUR NUMBERS

16 5 2 9

11 3 7 7

8 4 0 4

4 4 4 4

0 0 0 0

The pattern starts with the four numbers: 16, 5, 2, 9
Numbers next to each other are subtracted to make the new row: 11, 3, 7, 7
The 7 on the right-hand end is obtained by finding the difference between the first and last number.
The 3rd row is obtained from the 2nd row in the same way, and so on.

No matter what four numbers are chosen the pattern produced will always end in a row of zeros.

Even if very large and very small numbers are used, the pattern will often reach a row of zeros after surprisingly few steps.

6 241 17 964

235 224 947 958

11 723 11 723

712 712 712 712

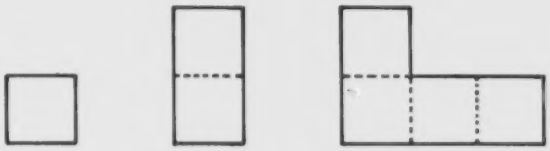
0 0 0 0

This activity gives plenty of subtraction practice. An additional benefit is that pupils need to consider which way round each subtraction has to be performed.

CHALLENGE: Who can find a pattern with the most rows?

* * *

LINE SYMMETRY



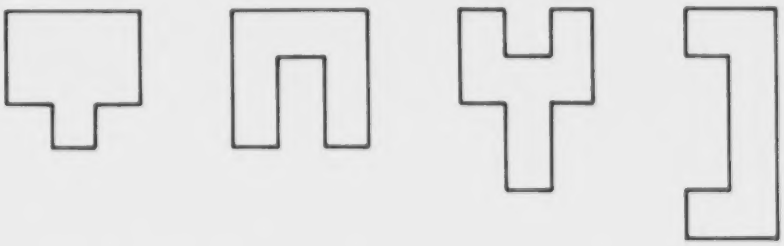
Fit the three pieces together to make a shape that has line symmetry.
How many different shapes are possible?

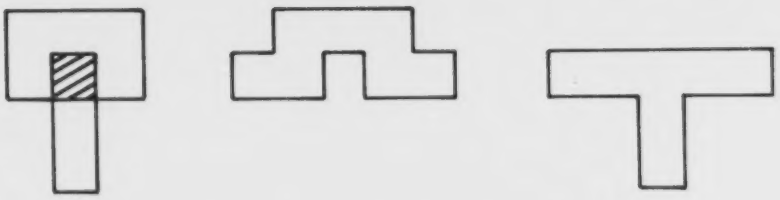
This activity can be used to introduce or reinforce some of the basic ideas of line symmetry.

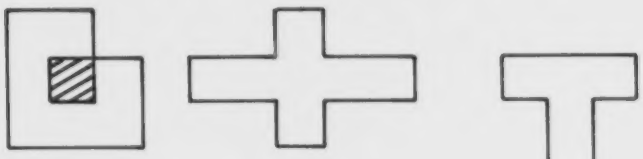
Children will need a set of the shapes, squared or 'dot' paper for recording answers and a mirror for checking.

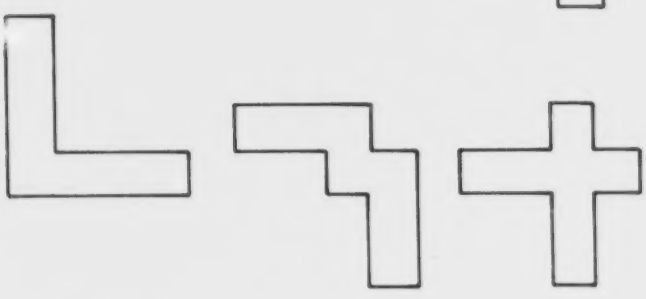
As the activity proceeds, questions will arise and decisions will need to be made about the meaning of 'fitting the pieces together'. This is a good opportunity for children to discuss the possibilities and maybe make their own rules for acceptability. This gives them a sense of control over the activity and a greater involvement in it.

If the pieces are placed so that they fit exactly along common edges then 15 different shapes can be made. (I think!)

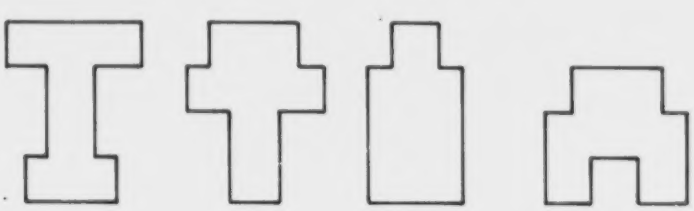


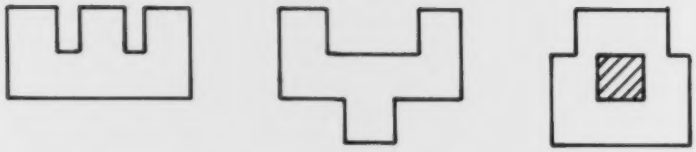




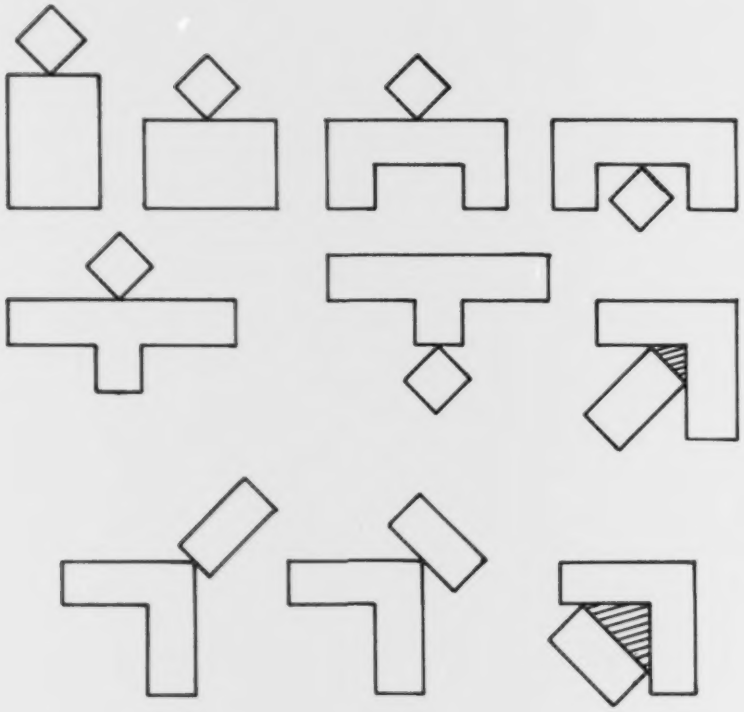


If the 'exactly along a common edge' rule is relaxed and pieces are allowed to fit anywhere along a common edge then 7 more shapes are possible.





A further 10 shapes can be made if the pieces are allowed to be joined at a corner.



Several similar activities, together with soft plastic shapes, reflection boards and teachers notes on line symmetry, are available in a 'Symmetry Puzzle Pack' compiled by Geoff Giles - University of Stirling.

For more information contact your local mathematics workshop.

* * *

INVESTIGATION

Take any 2-digit number: E.g. 74

Multiply the digits together. 7 x 4 = 28

Multiply the digits of the new number. 2 x 8 = 16

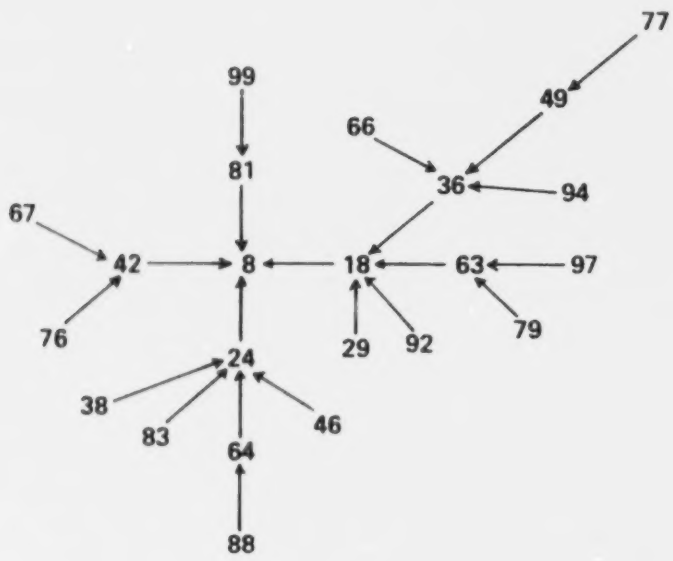
Continue to multiply until a single digit number is obtained: 1 x 6 = 6

Form a chain: 74 → 28 → 16 → 6

What other 2-digit numbers end at 6?
Link them together in one diagram.
How do you know you have found them all?

Investigate to find out where all 2-digit numbers end up.

This diagram shows the numbers which end at 8.



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SORTING GRIDS FOR ATTRIBUTE MATERIALS

Many classrooms have a set of attribute blocks consisting of:

5 shapes	:	square, rectangle, circle, hexagon, triangle
3 colours	:	red, yellow, blue
2 sizes	:	large, small
2 thicknesses	:	thick, thin

a total of 60 pieces.

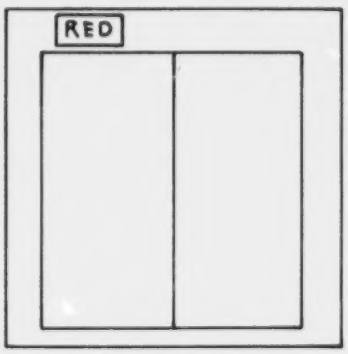
The blocks are used in the infant and lower primary grades for sorting activities, 'difference' games and spatial work associated with two and three dimensional shapes.

The sorting grids described below can be used in conjunction with the attribute blocks to provide some good thinking exercises.

Most of the grids need to be at least 40cm square. A set of labels is required denoting the various attributes: thick, thin, red, yellow ... etc. These are used to define what must be placed in a given row or column and by implication what cannot be placed.

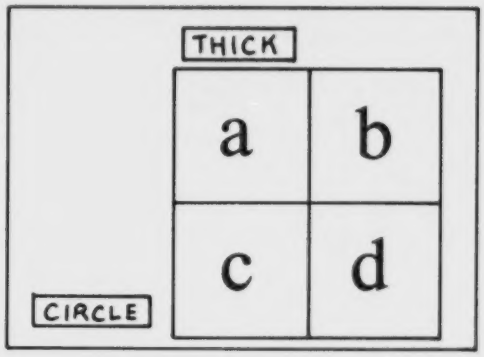
One example of each of 4 grids is described. Several more examples can be constructed by changing the labels.

1 - attribute grid



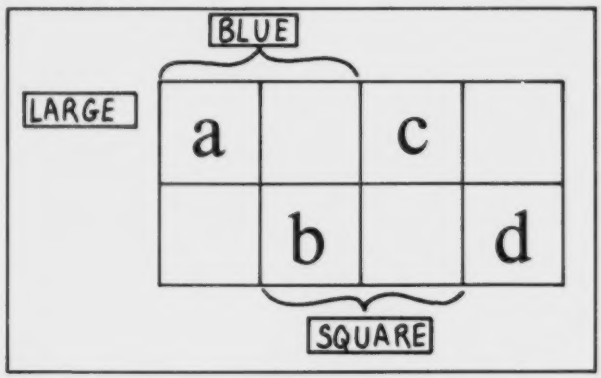
All red shapes must be placed in the first column. Shapes which are not red are placed in the second column.

2 - attribute grid



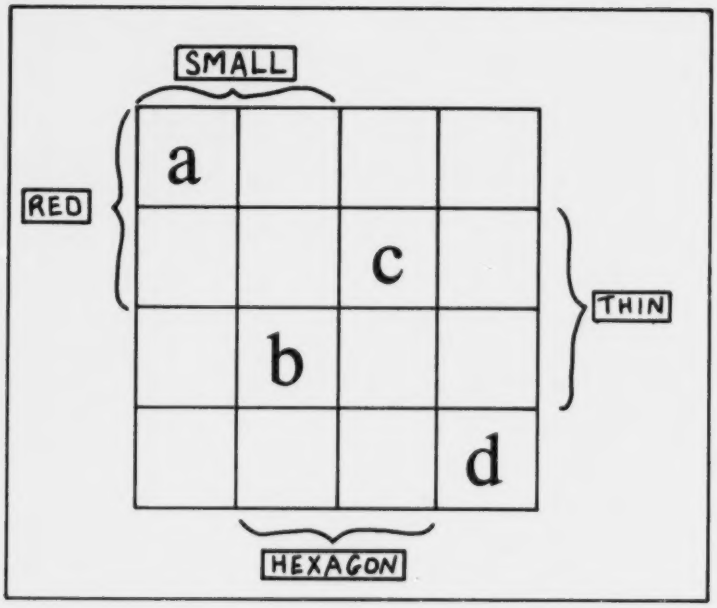
In (a) shapes which are thick and not circles
(b) shapes which are not thick and not circles
(c) shapes which are thick and circles
(d) shapes which are not thick and circles

3 - attribute grid



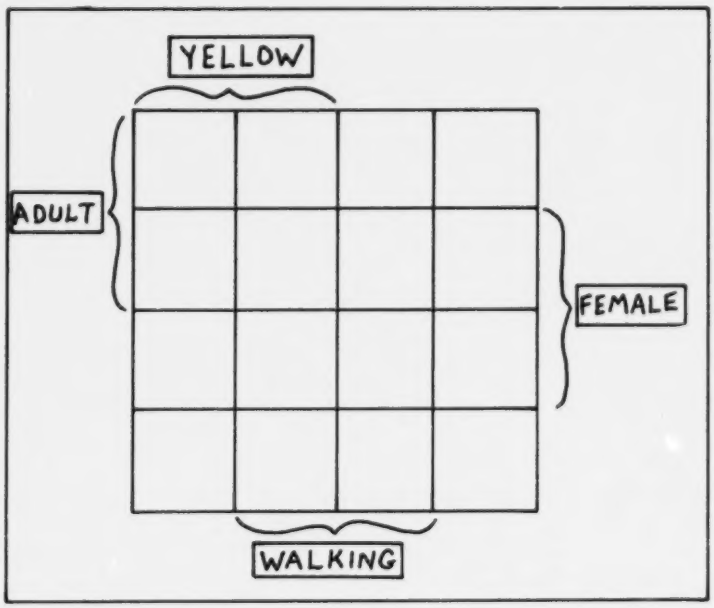
In (a) shapes which are blue, large and not square
(b) shapes which are blue, not large and square
(c) shapes which are not blue, large and square
(d) shapes which are not blue, not large and not square

4 - attribute grid



In (a) shapes which are small, red, not thin and not hexagons
(b) shapes which are small, not red, thin and hexagons
(c) shapes which are not small, red, thin and hexagons
(d) shapes which are not small, not red, not thin and not hexagons

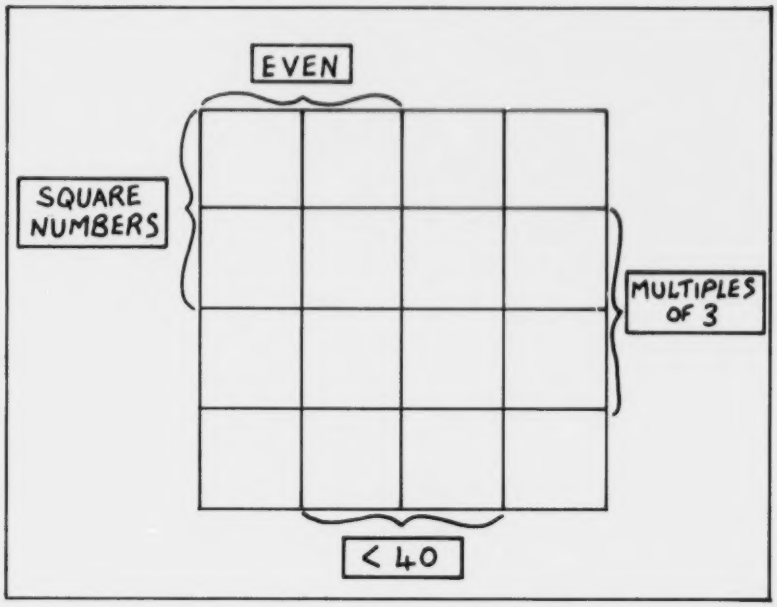
The grids can be used with other attribute materials e.g. logic people.



Make a set of 16 cards for the numbers

9, 10, 11, 15, 16, 24, 25, 36, 44, 45, 49, 59, 60, 81, 100, 144.

Place them in their correct position on the grid below.



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How many of the counting numbers 1, 2, 3, 4, 5 ... 150 contain the digit 3?

*

When John lined up his toy soldiers in 2's there was one over. There was also one over when he lined them up in 3's, and again when he lined them up in 4's and also 5's.

What is the smallest number of toy soldiers that John could have had?

*

MAGIC SQUARES

The following activities all involve constructing and using number squares. To the extent that they produce results which are unexpected and quite extra-ordinary, they could all be called Magic Squares.

A very important point here is that it is the way we present them to children and the enthusiasm that we impart that really makes them magic.

Number squares, which traditionally are called Magic Squares, are those in which the sum of the numbers along each row, column and diagonal is the same. They are usually forned using a set of consecutive numbers and are classified according to the number of cells along each side of the square.

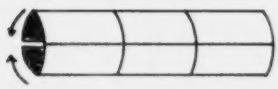
Magic Squares with an odd number of rows and columns can easily be constructed. Details are given below.

Consider the construction of a 3 x 3 and a 5 x 5 Magic Square using the numbers 1 to 9, and 1 to 25 respectively. The numbers are placed in position in ascending order.

RULE 1: Place 1 in the middle cell of the top row.

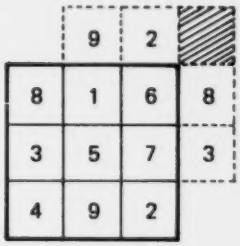
RULE 2: Place each successive number diagonally upwards to the right. (one square to the right and one square up).

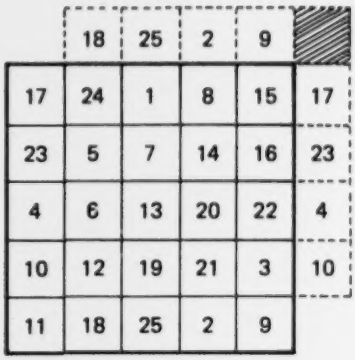
RULE 3: You must visualise the top and bottom lines of the square as being attached to each other - as a cylinder, also the left and right sides in the same way.



If rule 2 places a number outside the square at the top it will be transferred to the bottom. Likewise if a number is placed outside the square to the right it will be transferred to the extreme left.

RULE 4: If the cell you wish to use is already occupied, that number is placed immediately below the previously placed number. This rule also applies for the shaded square (see diagrams below).





Notice that for the 3 x 3 square the total of the rows, columns and diagonals is 15 and this is three times the number in the central position (5). For the 5 x 5 square the total is 65, which is five times the number in the central position (13).

The number in the central position is the middle number in the set of numbers used for the square.

1, 2, 3, 4, 5, 6, 7, 8, 9.

1, 2, 3, ... 13 .....23, 24, 25.

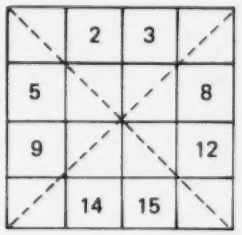
If you wanted to construct a 7 x 7 Magic Square using the numbers 1 to 49, then the middle number will be 25 and the magic total 7 x 25 = 175. Using the rules the square can be constructed and these facts checked.

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

The rules for constructing squares with an even number of cells along each side are more complex and I will only give those for a 4 x 4 square. (The 2 x 2 square is impossible).

RULE 1: Draw in both diagonals.

RULE 2: Start at the top left hand corner and work to the bottom right hand corner counting as you go. Whenever you come to a cell which has the diagonal passing through it, you miss out that cell.



RULE 3: Start again at the top right hand corner and fill in the 'diagonal squares'. Count backwards from 16.

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

This is a special Magic Square, for not only do all the rows, columns and diagonals add up to 34 but also each corner block of four numbers and the central block of four numbers have a sum of 34.

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

11	10
7	6

The above is only an introduction to what there is to know about the fascinating mathematics of Magic Squares. If you would like more information refer to the books listed at the end of this article.

Place the consecutive numbers 1, 2, 3, ... into a square array such that all the rows, columns and diagonals have different sums.

A 2 x 2 square is impossible but other squares are not difficult to find.

9	8	7	= 24
2	1	6	= 9
3	4	5	= 12
11	14	13	18 15

1	2	3	4	= 10
5	6	7	8	= 26
9	10	11	12	= 42
13	14	16	15	= 58
34	28	32	37	39 33

A more challenging problem is to add the further condition that the different sums formed must also be part of a set of consecutive numbers.

These kind of squares are Antimagic Squares.

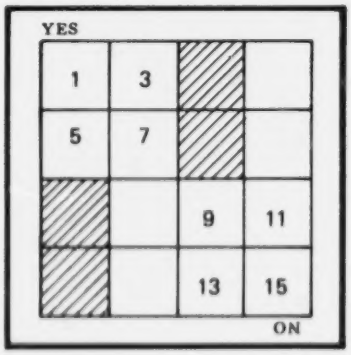
2 x 2 and 3 x 3 squares have been proved impossible, but there are many different solutions to the 4 x 4 and larger squares.

10	14	2	4	= 30
1	9	16	7	= 33
13	3	8	15	= 39
12	6	5	11	= 34
35	36	32	31	37	38

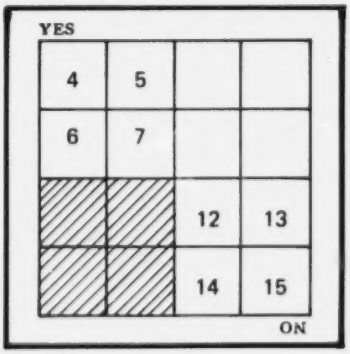
For the next activity you will need four pieces of squared paper or card, each having a side length of 6 centimetres.

Outline a 4 x 4 square in the centre of each card leaving a 1cm margin all around. Copy the pattern for each card and cut out very carefully the shaded rectangles. Notice that card 4 has numbers written on the back as well as the front. This must be done carefully and is carried out by turning the card over (reflection) about the dotted line.

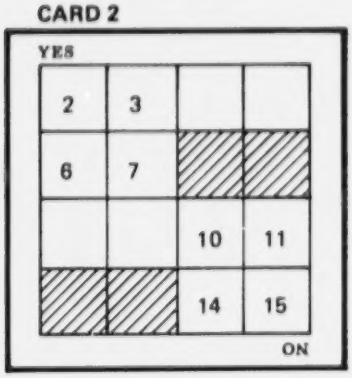
CARD 1



CARD 3



CARD 2



CARD 4 (FRONT)

YES

8	9		
10	11		
		12	13
		14	15

NO

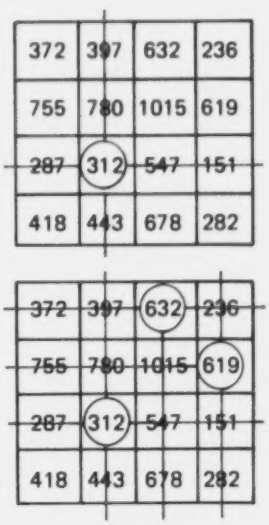
CARD 4 (BACK)

8	9	5	7
10	11	4	6
1	3	12	13
	2	14	15

When the cards are ready, ask someone to think of a number from 1 to 15. Show him the front of each card in turn, and ask him if his number is on the card. If he says 'yes' put the card down on the table with the word 'yes' on top. If he says 'no' turn the card so that the 'no' will be on top. The cards should be placed one on top of the other. Cards 1, 2 and 3 may be used in any order but card 4 must be the last card that you show and is placed on top of the pile. Pick up the pile of cards and turn them over. The correct number will show through a window in the cards.

---

1. Circle any number in the square.
2. Draw a line through the row and column containing this number. When a line has been drawn through a number this cannot be selected at a later stage.
3. Circle two more numbers and draw the lines. One number will remain which does not have any lines through it.
4. Find the sum of this number and the three numbers you have circled.

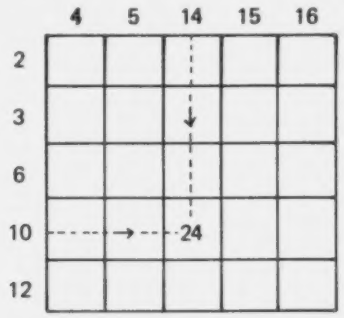


418 + 312 + 632 + 619 =

The answer will always be 1981!

The squares can be of any size and you can make any number you choose the answer.

1. Choose the size of the square, say 5 x 5.
2. Choose the number, say 87.
3. Write down any ten numbers that have a sum of 87,
   e.g. 2, 3, 4, 5, 6, 10, 12, 14, 15, 16.
4. Divide these ten numbers arbitrarily into two sets of five.
   $\{2, 3, 6, 10, 12\}$ $\{4, 5, 14, 15, 16\}$
5. Write the first set of five numbers vertically beside the first column of the square and the second set horizontally above the first row.



6. Add the numbers in corresponding rows and columns to produce the table.

6	7	16	17	18
7	8	17	18	19
10	11	20	21	22
14	15	24	25	26
16	17	26	27	28

I'm sure you will agree that the above activities provide a challenge and a tremendous amount of very good number work. In the last example above, for instance, it would be very interesting to see how children would go about selecting a set of ten numbers which have a sum of 87.

For further information see:

Creative Puzzles of the World by Pieter van Delft and Jack Botermans, Cassell.

Magic Squares and Cubes by W S Andrews, Dover.

Madachy's Mathematical Recreations by J S Madachy, Dover.

6	14	44
26	1	54
4	10	2

MAGIC SQUARE!

* * *

The sum of 4 numbers is 45.

If 2 is added to the 1st number; and

2 is subtracted from the 2nd number; and

the 3rd number is multiplied by 2; and

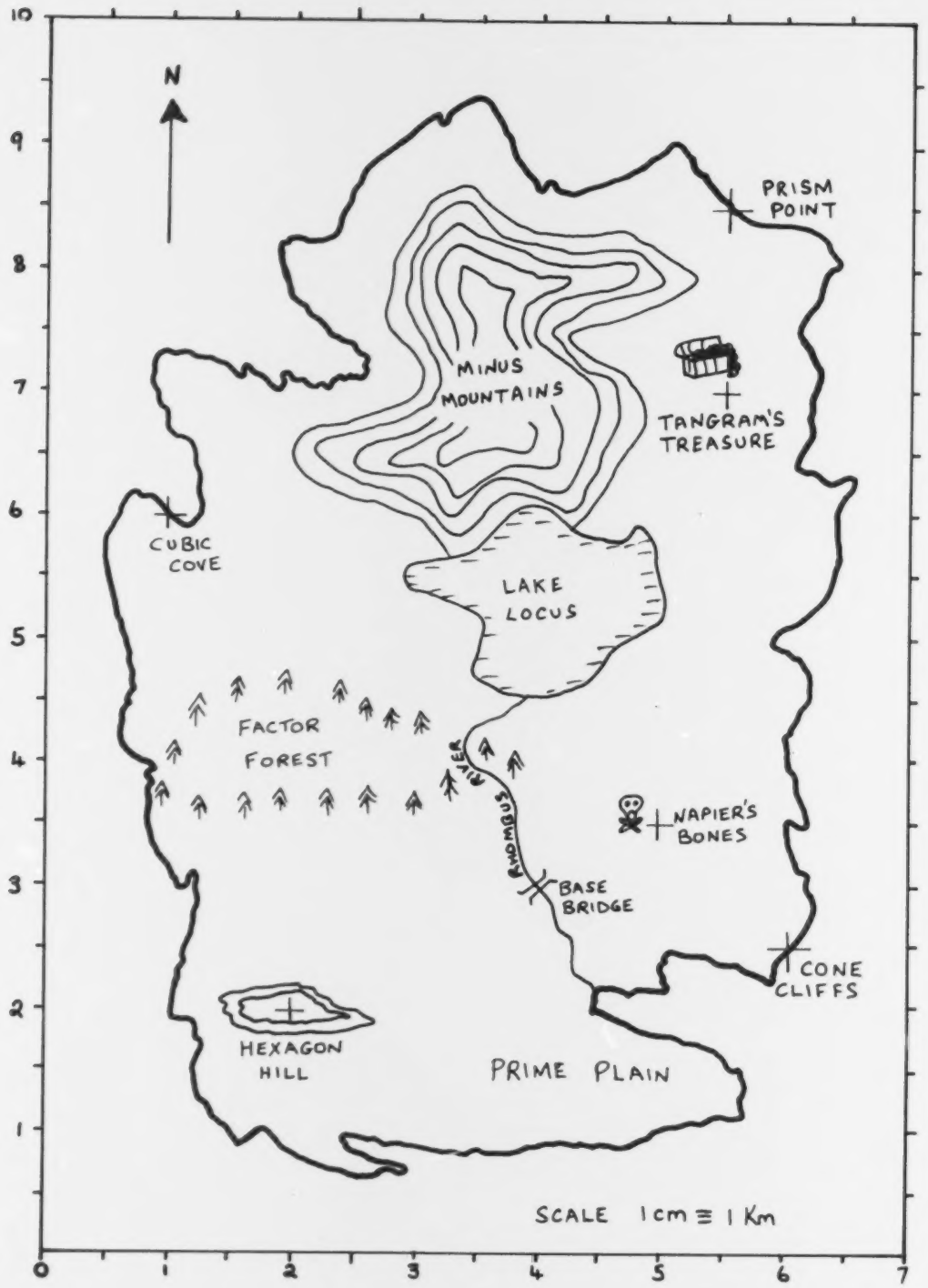
the 4th number is divided by 2,

then the result of each calculation will give the same answer.

What are the four numbers?

*

INTEGER ISLAND



1. What places on the map have the coordinates:
   (4, 3) (5.5, 7) (6, 2.5)?

2. Write down the coordinates of: Cubic Cove, Hexagon Hill, Napier's Bones and Prism Point.

3. Calculate the shortest distance between:
   a) Prism Point and Tangram's Treasure
   b) Hexagon Hill and Base Bridge
   c) Cubic Cove and Hexagon Hill
   d) (2, 4) to (4, 2)
   e) (3.5, 6.5) to (3, 3)

4. What is the distance from east to west of Lake Locus at its widest point?

5. Name all the places on the map which are within 7 km of Base Bridge.

6. Calculate the perimeter of the island.

7. If a ship circumnavigates the island, always keeping approximately 1km from shore, how far does it travel?

8. Find the approximate area of the island.

9. I start at Hexagon Hill and go 6km north and 4km east. Where am I?

10. I start at Prism Point and go 9km south and 7km west. Where am I?

11. Write down the bearing of:
   a) Base Bridge from Cubic Cove;
   b) Tangram's Treasure from Hexagon Hill;
   c) Napier's Bones from Cone Cliffs.

12. What are Napier's Bones?

13. Captain Polygon landed at Cubic Cove and went in search of Tangram's Treasure. Describe his journey.

* * *

SO YOU STILL DON'T KNOW YOUR TABLES!

Set out below is a method that can be used to motivate some children in the upper part of the primary school to learn the basic multiplication facts. If it works for one child - it is worth it.

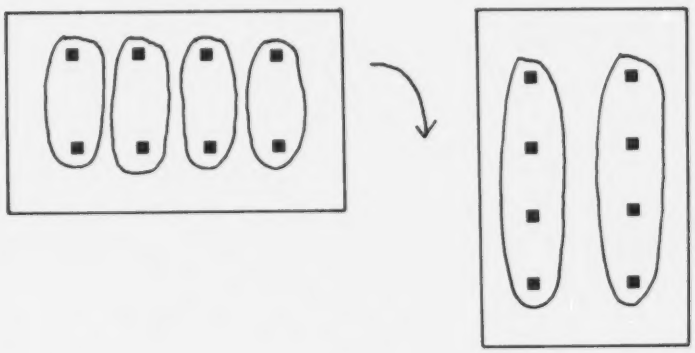
- How many multiplication facts do you need to know?

  You need to be able to multiply any number 0, 1, 2, ... 10 by any other number 0, 1, 2, ... 10.

  That is 121 (11 x 11) facts to learn.

  No wonder you can't remember some of them!

- Multiplication is commutative i.e.
  4 x 2 = 2 x 4, 8 x 3 = 3 x 8, ... etc.



If you know one fact you also know the corresponding one.
There are 55 of these products.
You have only 66 facts to learn.

×	0	1	2	3	4	5	6	7	8	9	10
0	0										
1	0	1									
2	0	2	4								
3	0	3	6	9							
4	0	4	8	12	16						
5	0	5	10	15	20	25					
6	0	6	12	18	24	30	36				
7	0	7	14	21	28	35	42	49			
8	0	8	16	24	32	40	48	56	64		
9	0	9	18	27	36	45	54	63	72	81	
10	0	10	20	30	40	50	60	70	80	90	100

- Any number multiplied by 0 equals 0.
  $0 \times 0 = 0, \quad 1 \times 0 = 0, \ldots 10 \times 0 = 0.$

  There are 11 of these products.

  You have only 55 facts to learn.

- Any number multiplied by 1 is equal to the number itself.
  $1 \times 1 = 1, \quad 2 \times 1 = 2, \ldots 10 \times 1 = 10.$

  There are 10 of these products.

  You have only 45 facts to learn.

- Multiples of 10.
  Study the pattern - numbers move to the left; add a zero.
  $2 \times 10 = 20, \quad 3 \times 10 = 30, \ldots 10 \times 10 = 100.$

  Also, in a mechanical way, if you know:

$$\begin{array}{r} 0 \\ \times\ 7 \\ \hline 0 \end{array} \quad \text{and} \quad \begin{array}{r} 1 \\ \times\ 7 \\ \hline 7 \end{array} \quad \text{then you know} \quad \begin{array}{r} 10 \\ \times\ 7 \\ \hline 70 \end{array}$$

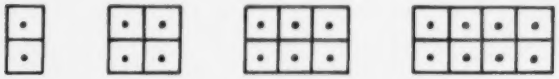
  There are 9 of these products left.

  You have only 36 facts to learn.

- Multiples of 2.

  Study the pattern of even numbers.

  2 x 2 = 4, 3 x 2 = 6, ... 9 x 2 = 18.



  There are 8 of these products left.

  You have only 28 facts to learn.

- Multiplies of 5.

  Study the pattern: 0, 5, 0, 5, 0, 5 ...

  Relate this to the multiples of 10.

  3 x 5 = 15, 4 x 5 = 20, ... 9 x 5 = 45

  There are 7 of these products left.

  You have only 21 facts to learn.

X	0	1	2	3	4	5	6	7	8	9	10
0	0										
1	0	1									
2	0	2	4								
3	0	3	6	•							
4	0	4	8	•	•						
5	0	5	10	15	20	25					
6	0	6	12	•	•	30	•				
7	0	7	14	•	•	35	•	•			
8	0	8	16	•	•	40	•	•	•		
9	0	9	18	•	•	45	•	•	•	•	
10	0	10	20	30	40	50	60	70	80	90	100

3 x 3	4 x 4	6 x 6	7 x 7	8 x 8	9 x 9
4 x 3	6 x 4	7 x 6	8 x 7	9 x 8	
6 x 3	7 x 4	8 x 6	9 x 7		
7 x 3	8 x 4	9 x 6			
8 x 3	9 x 4				
9 x 3					

If you learn one fact a day then you can complete the task in 3 weeks!

There are a number of important questions that need to be considered in connection with the teaching and learning of multiplication facts.

E.g.
- Should we teach 'tables'?
- Do children need to memorise 11 x and 12 x facts?
- What degree of fluency in multiplication do we want children to acquire?
- Is the memorisation of facts more important than the understanding of them?
- What choice of methods do we have for teaching multiplication?
- What place, if any, does the electronic calculator have in the teaching and learning of multiplication facts?

* * *

If a whole number less than 20 is divided by another whole number less than 20 the result is 2.2.

What are the two numbers?

*

Two frogs have a race. Kermit makes a jump of 80cm once every 5 seconds. Hermit makes a jump of 15cm every 1 second. The rules of the race state that the frogs must cross a line 5 metres away and then come back to the starting point.

Which frog wins the race and how long does he take?

*

The sum of two whole numbers is 215.

If the smaller number is subtracted from the larger, the result is 37.

What are the two numbers?

*

MONEY SNAKES

Materials required: 16 cardboard squares
plastic coins
glue

The cardboard squares need to be at least 9cm by 9cm. If you want to make them larger, choose the side length to be a multiple of 3cm - it makes it easier for construction purposes.

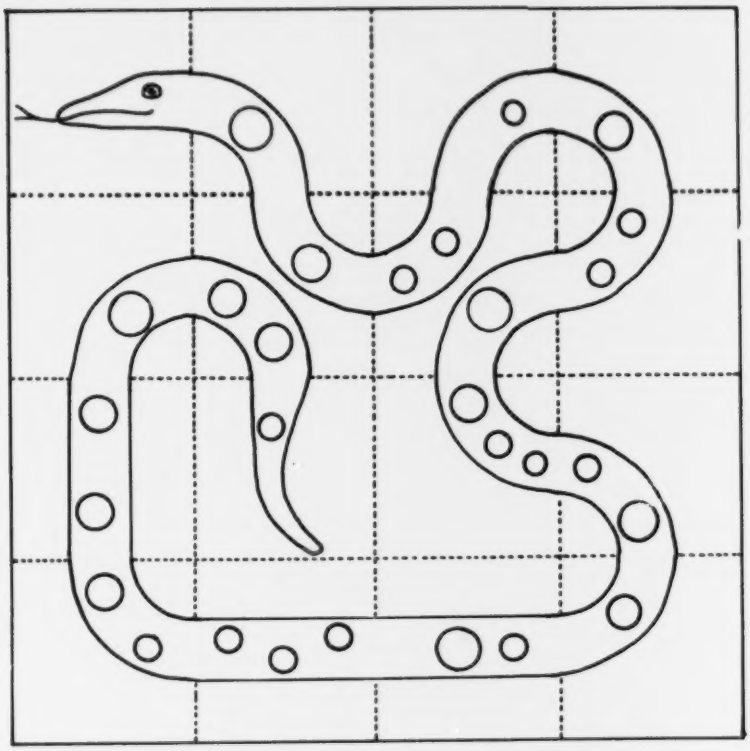
Snake sections are drawn on the squares. It doesn't matter what proportion of curves to straights you have. You will need only one head and a tail section.

Because the squares have to be fitted together in different ways to form snakes, it is important that the snake sections are drawn from, and to, the middle third of each side.

When the snake sections have been drawn, paste plastic coins on them.

The cards can be used in a variety of ways and provide an opportunity for some elementary problem solving.

E.g.
- Make a snake.
- Make a 20¢ snake.
- Make a 50¢ snake.
- How many different snakes can you make having a value of 25¢?
- If you use all the pieces to make a snake, how much is it worth?
- Make a snake so that the pieces form a 3 x 3 square, 4 x 4 square, 4 x 3 rectangle.



* * *

$17 \times 23 = 391$

Use this result to find the answers to the following:

$391 \div 23 =$	$34 \times 23 =$
$\frac{1}{17}$ of $391 =$	$170 \times 230 =$
$18 \times 23 =$	$3910 \div 17 =$
$17 \times 22 =$	$1.7 \times 2.3 =$

*

Place the numbers: 1, 2, 3, 4, 5 in the squares - one number to each square, so as to make a true statement.

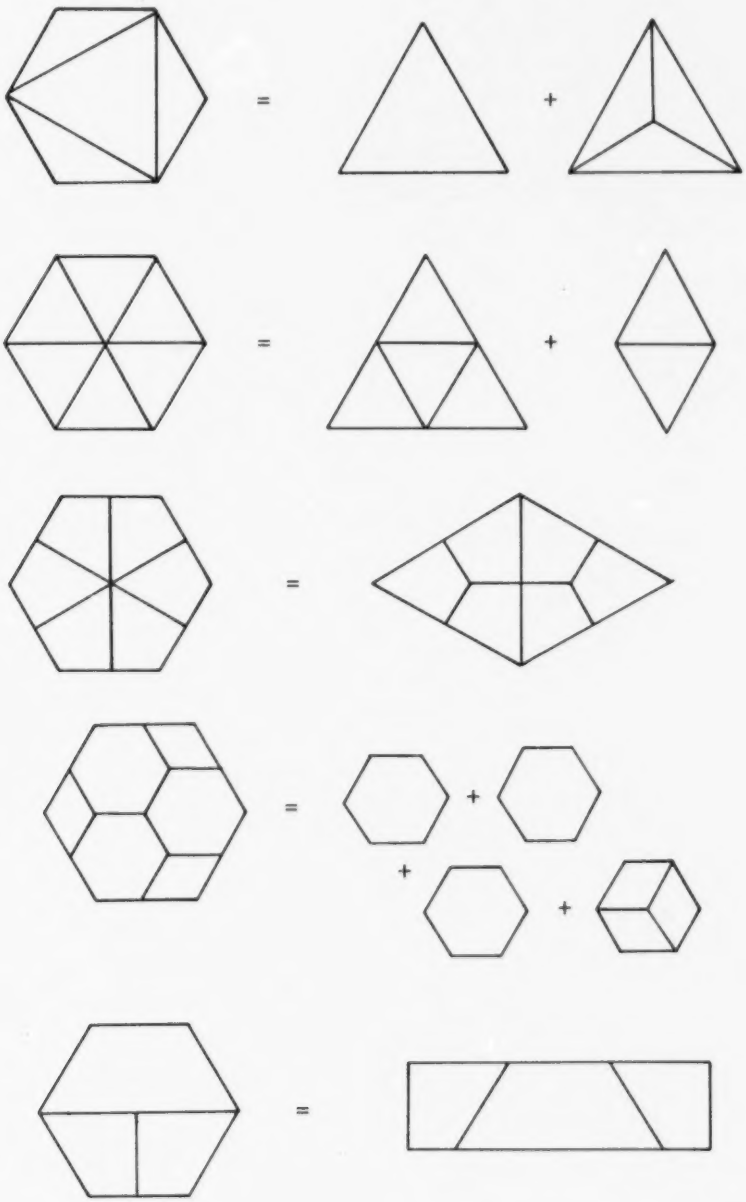
$\square \times \square + \square = \square\square$

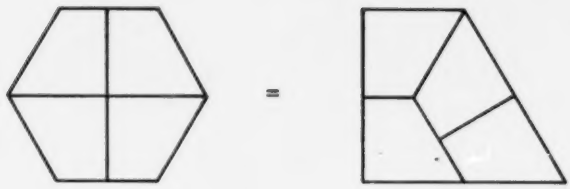
*

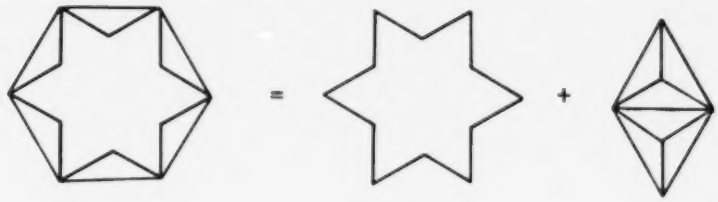
DISSECTING HEXAGONS

The diagrams below show a few of the ways in which a regular hexagon can be cut up and the pieces arranged to make further hexagons, triangles, rectangles etc.

Children can be encouraged to investigate the various possibilities and also try to reform the hexagon from the pieces.







This activity could be used as an introduction to some challenging work involving area and fractions. For example:

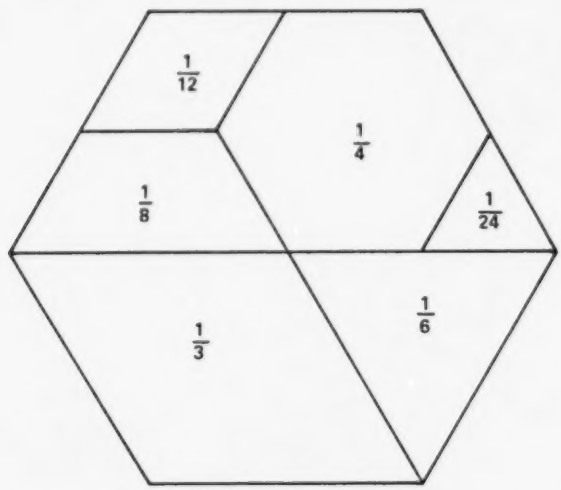
Find some shapes which have an area which is exactly $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ ... of the hexagon.

How many different shapes can you find for each fraction?

Do these shapes have the same perimeter?

Are there any fractions for which it is impossible to find a simple shape?

What about $\frac{2}{3}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{7}{8}$ ... ?



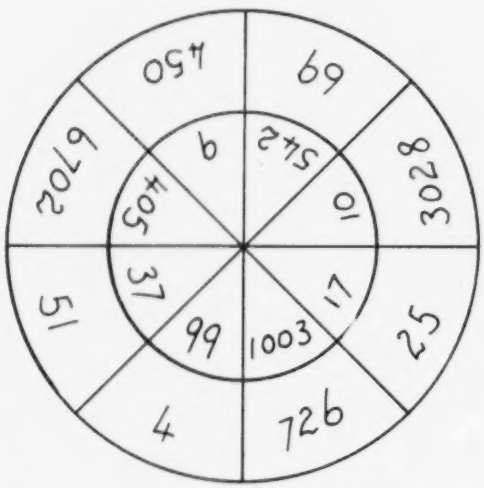
$\frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{12} + \frac{1}{24} = 1$

DIAL A PROBLEM

- Cut out 2 cardboard discs with diameters approximately 10cm and 7cm.
- Divide each disc into 8 equal sectors.
- Fix them together so that the smaller disc will rotate freely.
- Choose 2 sets of 8 numbers within the appropriate range for the child and write these on the discs.

The discs are used to generate sets of number problems (+, -, x, ÷) and can be used by individuals, groups or whole classes. Children should have their own discs.

Every number on the outer disc can be matched with a number on the inner disc to produce 64 different pairs of numbers, and hence, 64 different addition, subtraction, multiplication and (for grade 6) division problems. For subtraction problems it will be necessary to give the instruction, 'Subtract the smaller number from the larger number'.
For division problems, 'Divide the larger number by the smaller number. Give your answer correct to 1 decimal place'.



The discs shown could be used by children in upper primary classes. The numbers have been chosen carefully.
They include:

- 1, 2, 3 and 4 digit numbers;
- zeros in different place-value positions specifically for subtraction problems;
- 99, 25 and 10 so that mental methods can be employed.

Consider the 8 pairs of numbers matched on the discs shown.
(99, 4) (1003, 726) (17, 25) (10, 3028) (542, 69) (9, 450) (405, 6702) (37, 51)

Children should be encouraged to make decisions about the pairs of numbers before they start to calculate.

DECISION: Which problems can be done mentally?

\+ Perhaps most of them, but certainly:
99 + 4, 3028 + 10, 450 + 9

\- 99 - 4, 25 - 17, 3028 - 10, 450 - 9
If subtraction is thought of as 'adding-on' then
1003 - 726 = 274 + 3 and
51 - 37 = 13 + 1

x 4 x 99, 17 x 25, 10 x 3028
N.B. 4 x 99 looks easier than 99 x 4 and
17 lots of 25 is easier than 25 lots of 17

÷ Certainly 3028 ÷ 10 and 450 ÷ 9, perhaps also 99 ÷ 4

DECISION: For those problems that have to be written down - how should the figures be placed in relation to each other?

6702	542	542	$17\overline{)25}$
+ 405	- 69	x 69	

To set problems out in this way requires decisions to be made about

place-value, the relative 'size' of the numbers involved and a working knowledge of the 'process'.

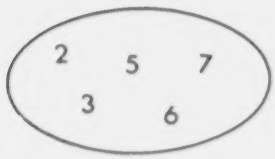
E.g. $17\overline{)25}$ is the same as 25 ÷ 17

DECISION: Are the answers reasonable?
Children should be encouraged to:

ESTIMATE ⟶ CALCULATE ⟶ CHECK

Move the centre disc one place to the left or right and 8 more problems appear.

* * *



Select two numbers from those given and form a fraction equivalent to:

a) $\frac{10}{12}$ b) $\frac{1}{2}$ c) $1\frac{1}{2}$ d) $2\frac{1}{3}$

e) 3 f) 0.6 g) 1.4 h) 40%

*

3, 6, 14, 17, 19, 25, 32, 46, 51

Share these numbers into:

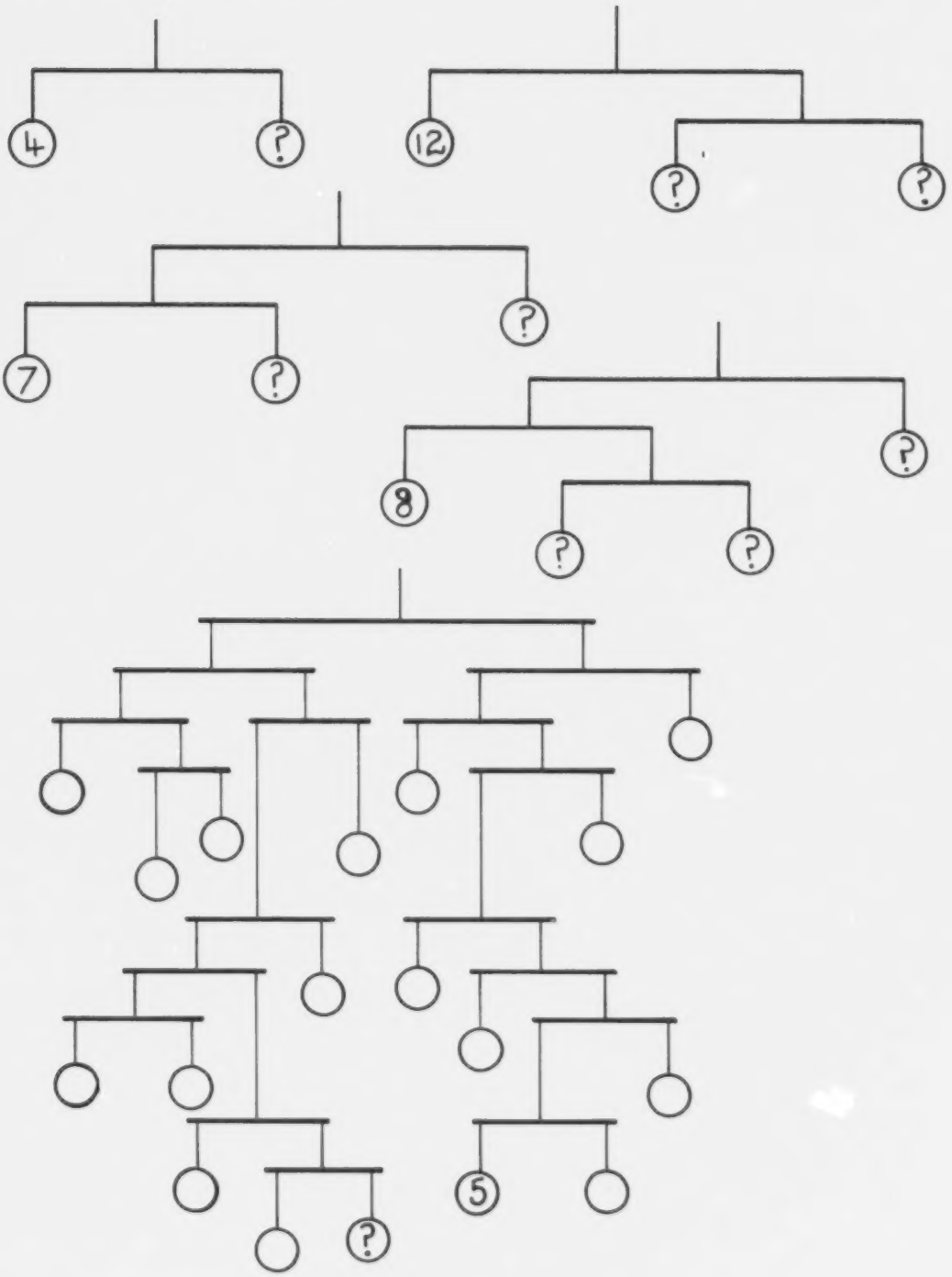
a) 3 sets, so that the sum of the numbers in each set is the same.

b) 2 sets, so that the difference between the sum of the numbers in each set is as small as possible.

*

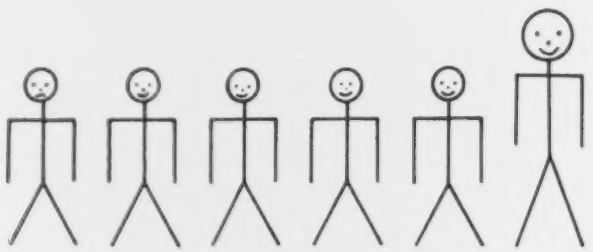
MOBILE MATHS

(Assume the rods have no value).



The Human Computer

Five children stand in a line facing the rest of the class. They should space themselves so that their right hand can just reach the next person's left shoulder. These people form the 'calculating unit' and 'output' of the machine. The teacher acts as the 'input unit' and also faces the class on the far left of the line.



Instructions for each 'cog' in the machine:

a) Only your right hand is allowed to move.
   Your left hand remains at your side.

b) Your right hand can either be UP or DOWN.

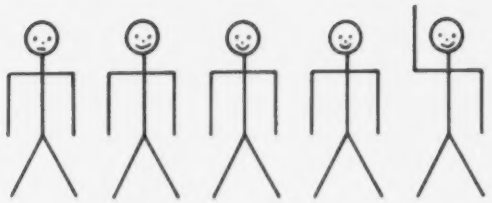
c) When you receive a tap on your left shoulder you alter the position of your right hand.
   If it was UP you bring it DOWN; if it was DOWN you put it UP.

d) Everytime you bring your hand DOWN you tap the next person on the left shoulder.

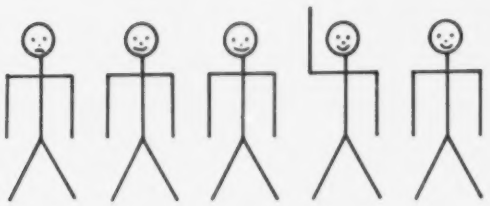
e) For the starting position all hands are down.

To set the machine in motion the teacher taps the first boy on his left shoulder. The teacher continues to tap very slowly and the rest of the class count the number of taps.

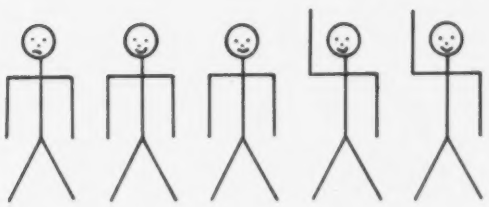
1 =



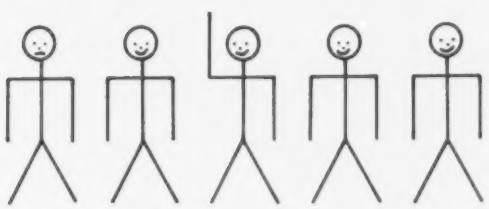
2 =



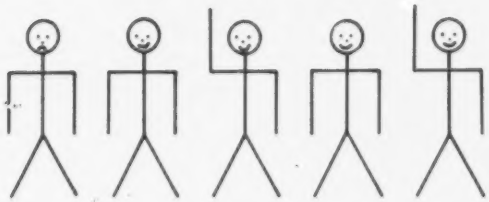
3 =



4 =



5 =



It will need quite a bit of practice before the machine runs smoothly. Several of the cogs will need oiling!

How does the machine count?

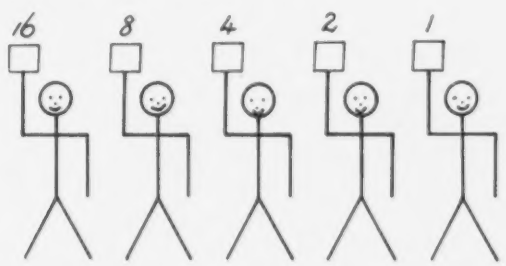
The class can record the 'output' of the machine in the form of a table. A hand UP can be denoted by a 1, a hand DOWN by 0.

NUMBER					
1					1
2				1	0
3				1	1
4			1	0	0
5			1	0	1

What number can the machine count up to?
How does the machine show 17, 23, 31?
How could the machine count up to larger numbers?

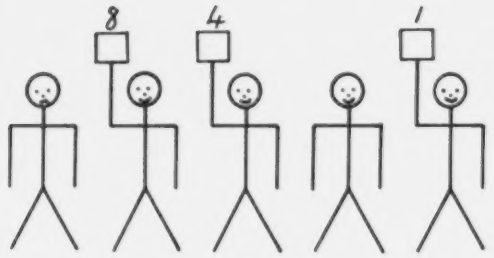
Give each person in the machine a card.
The card should have a piece of elastic threaded through it so that it can slide on to the right hand.

The cards should be numbered as shown below:



The machine starts again and this time when a person raises his arm he also displays a card.

E.g.

13 = 

Children can take it in turns to call out a number and the machine has to display that number straight away without having to count from 1.
(No talking allowed).

Binary Arithmetic

Following on from the activities described above children can be encouraged to think more deeply about the counting system which has been developed using the machine. It is fairly obvious that we are operating in base 2 and that the place value assigned to each 1 or 0 is a power of 2. This is known as the BINARY SYSTEM.

←····	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
←····	128	64	32	16	8	4	2	1

The example below shows one way of converting a number (base 10) to a binary number.

E.g. 105

Starting from the left 2^7 and higher powers are	105
too large.	- 64
We need one 64, which when subtracted from	41
105 leaves 41.	- 32
We need one 32, which leaves 9.	9
No. 16 required, one 8, no 4, no 2, one 1 -	- 8
giving	1
1101001	- 1
	0

To change from binary to base 10 the opposite to the above process is carried out.

E.g. 10110101

Write down the column headings.								128
Place the numbers underneath and add up								32
the numbers corresponding to the 1's.								16
								4
128	64	32	16	8	4	2	1	1
1	0	1	1	0	1	0	1	= 181

Children could be asked to investigate how they would add, subtract, multiply and divide binary numbers.
The complete addition and multiplication tables are shown below.

+	0	1
0	0	1
1	1	10

x	0	1
0	0	0
1	0	1

```
  1 1 0 1      1 1 0 1        1 0 1 1
+ 1 0 1 1    -   1 1 0      x   1 1
-------      -------        -------
1 1 0 0 0      1 1 1        1 0 1 1 0
                          1 0 1 1
                          -------
                          1 0 0 0 0 1
```

```
            1 0 1
        -----------
1 0 1 ) 1 1 0 0 1
        1 0 1
        -----
          1 0 1
          1 0 1
          -----
```

It really is very easy!

Applications

Any number, however large, can be recorded by using only the two symbols 0 and 1. Because of this the binary system is widely used in electronic computers. The use of two symbols makes it possible to transmit numbers by the presence or absence of electrical pulses.

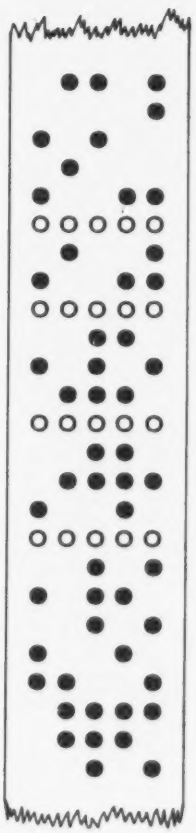
Codes

If each letter of the alphabet is assigned a numerical value (A = 1,

B = 2, C = 3,...) then letters can be represented in binary form:

	16	8	4	2	1
A					•
B				•	
C				•	•
D			•		
E			•		•

Decode the message shown below:



A Number Puzzle

a) Construct a set of cards like those shown below:

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

b) Ask a friend to think of a number less than 32.

c) Show him each card in turn and ask if the number is on it.

d) Add the numbers on the top left hand corner of all the cards for which the reply was 'yes'.

e) This total is the number your friend thought of.

Those children who have studied the binary system should have no trouble in finding out why this puzzle works. They could be asked to extend the range up to 64 and to decide what numbers should be added to the existing cards and what numbers must be put on a 6th card.

* * *

A book is lying open on a desk. To what pages is it opened if the product of the numbers on the facing pages is 8556?

*

THE REAL CLASSROOM IS OUTSIDE
get into it

Most of you will be familiar with the above statement. It is written on a large poster distributed to all schools some time ago and is decorating the walls in some staffrooms today.

Unfortunately, some teachers take the instruction literally. They take their classes into the school grounds or away to camp and then attempt to justify being there by doing a little bit of Mathematics, English, Science etc. However, the 'real world' is an extremely complex place and in it there is very little meaningful mathematics that children are capable of doing. What often happens is that children are asked to undertake activities which are artificial, which bear no relationship to previous work studied (inside) and which are a complete waste of time and effort.

At the other extreme, some teachers prefer to remain inside and teach mathematics as the manipulation of signs and figures according to certain rules. The only 'real world' experience that the children get are trivial 'story problems' - a reward for correctly mastering the formal algorithm using arbitrary numbers.

The main reasons for going outside are:

- Activities can be undertaken on a much larger scale than is possible in the classroom.
- Objects can be seen which may act as a stimulus for mathematical work.
- It is fun and can be highly motivating.

Three types of situation will arise:

a) Having learned something of the mathematics involved in the structured environment of the classroom we go outside to apply it or to look for examples of it. Often this will occur in problem solving situations.

   E.g. - Construct a shape which has an area of 1 hectare.

   - Determine which cars passing the school gate are breaking the

speed limit.

- Find the height of the school flagpole.
- Look for examples where triangles are used to give strength to a particular construction.
- Examine the patterns formed by bricks in a wall and paving slabs on a courtyard.
- Orienteering exercises.
- etc.

b) If you are outside for a particular reason then the need to do some mathematics may arise quite naturally.

E.g. A school picnic may involve:

- a count of heads to see everyone is present.
- the measuring out of a cricket pitch.
- considerations of time: time to get there - time there - time to get back.
- decisions about spending money.
- deciding on the number of sausages, bread rolls, cans of cordial etc. required.

c) Mathematical skills are often required when work in other subjects is being undertaken outside.

E.g. sorting, sampling, mapping, measuring, collecting and representing information etc.

Sometimes objects brought into the classroom can be a stimulus for mathematical work.

E.g. - spiders, moths ... symmetry.
- shells, pine cones ... spirals, Fibonacci sequence.
- honeycomb ... hexagons, tessellations.
- coins ... dates, value, circles.
- model cars ... scale, proportion.

By all means go outside and use things from the outside but have a good reason for doing so. The following questions need to be considered before an outdoor activity is embarked upon.

- What is the aim of the activity?
- Have the children sufficient (indoor) mathematical knowledge?

- What apparatus is required and how much is needed?
- How will the children be organised?
- How much time will the activity take?
- How much space will be needed?
- Do the children have to 'produce' something?
  (answers, chart, diagram, object, model)
- Will the groups share their findings with the rest of the class?
- Is there any indoor follow-up work?

* * *

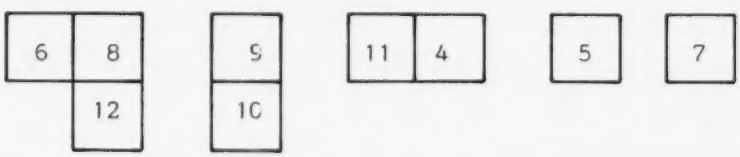
Take a long thin strip of paper and stretch it out in front of you, left to right.

Take the ends in your hands and place the right hand end on top of the left. Now press the strip flat so that it is folded in half and has a crease.

Repeat the whole operation on the new strip. Do it again and again and again ...

The paper will soon be getting too thick to bend. If it was possible to perform the operation outlined above a total of 10 times, how many creases would there be in the strip of paper?

*



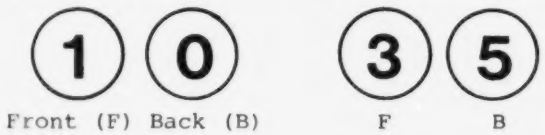
Put the pieces together to make a 3 x 3 square such that the sum of each row, column and diagonal set of 3 numbers is 24.

Draw a diagram of your arrangement.

*

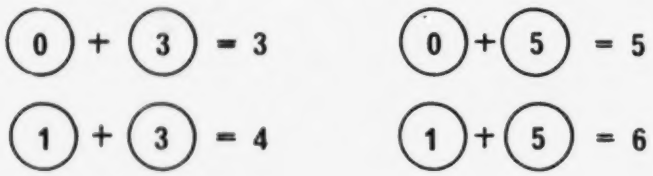
DISCS

Here are two cardboard discs.
A number is written on the front and back of each disc.



The discs are thrown up in the air and then the numbers on the upper-most faces are added together.

There are 4 possibilities:



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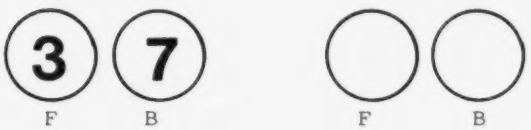
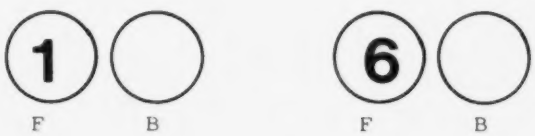
a)



Find the number on the front and back of the second disc if the totals are: 8, 11, 12, 15.

b)



Find the number on the back of each disc if the totals are: 3, 7, 11, 15.

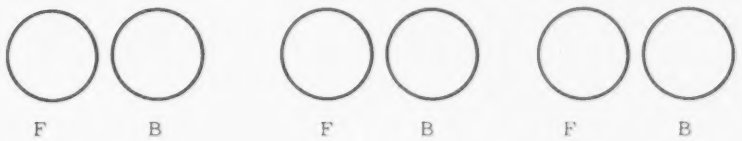
c)

What number is on the front and back of each disc if the totals are: 9, 10, 11, 12?

d) Suppose there are 3 discs.

What are the possible totals?
(Each time there are 3 numbers to add together.)

e)



What number is on the front and back of each disc if the totals are: 1, 2, 3, 4, 5, 6, 7, 8?

* * *

1, 2, 3, 4, 5, are consecutive numbers. They follow one after the other.

a) The sum of 4 consecutive numbers is 1442. What are the numbers?

b) The product of 3 consecutive numbers is 124,950. What are the numbers?

(You may use a calculator.)

*

REMAINDERS

I have always made the following two points when talking to teachers about remainders:

a) As children progress through the grades and learn new skills, the recording of a remainder is gradually refined as follows:

WHOLE NUMBER → FRACTION → DECIMAL

E.g. $17 \div 5 = 3R2 = 3\frac{2}{5} = 3.4$

b) The form of the remainder is determined by the question asked and children should be encouraged to make decisions about it.

E.g. (i) If I shared 53 marbles equally between 8 children, how many would each receive?
Answer: 6, with a remainder of 5.

(ii) If I bought 18 oranges to share equally between the 12 players in a team, how many would each receive?
Answer: $1\frac{1}{2}$

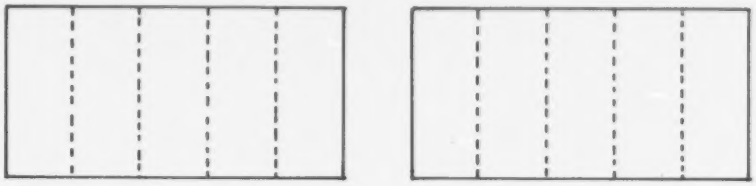
(iii) Calculate the batting average of a cricketer whose scores were:
16, 23, 4, 0, 17, 9, 34, 18, 20, 6.
Answer: 14.7

However, the word remainder means 'what is left after the division process has been carried out'.
E.g. $17 \div 5 = 3$ with $\underline{2}$ left over.

If the division process is applied to the remainder and the answer is expressed in fraction or decimal form, then there is nothing left over - no remainder. In the example above, if the 2 which is left over is divided by 5 the answer is $\frac{2}{5}$.

This is not immediately obvious.
Suppose the question was:
Share 17 chocolate bars equally between 5 children. How much does each receive?

Each child receives 3 bars and there are 2 bars left over.



The 2 bars are shared equally between the 5 children and each child receives $\frac{2}{5}$ of a bar.

Altogether each child receives $3\frac{2}{5}$ bars of chocolate and there is no remainder.

> Use a calculator to find
>
> 89328 ÷ 729
>
> What is the remainder?
> How can you use your calculator to obtain the remainder when you have done a division?

* * *

The counting numbers are arranged in 5 columns as follows:

1st	2nd	3rd	4th	5th
	1	2	3	4
5	6	7	8	
	9	10	11	12
13	14	15	16	
	17	18	19	20

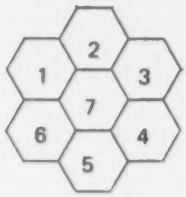
If the pattern continues on and on, in which column will the number a) 50, b) 100, c) 500, d) 1000 be?

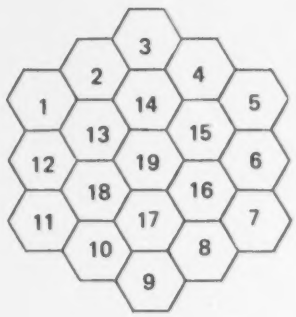
*

ACTIVITIES USING THE HEXAGON GRID

FREE PLAY - could lead to colouring, counting, numbering and discussing; involving ideas of pattern, symmetry, area, enlargement and tessellation.

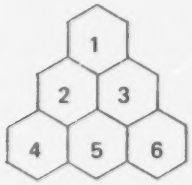
NUMBER PATTERNS

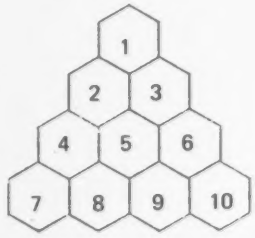




6 hexagons are needed to completely surround one hexagon. The total number of hexagons in the pattern being 7.
How does the pattern grow?
Is the total number of hexagons always a prime number?

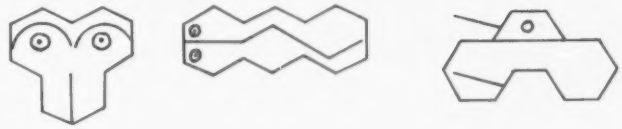
Experiment with other 'growth' patterns and see if you can predict the number of hexagons each time.
E.g.



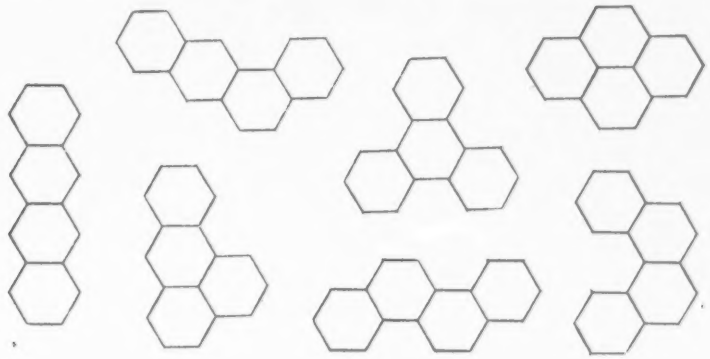


MAKING SHAPES - TESSELLATIONS

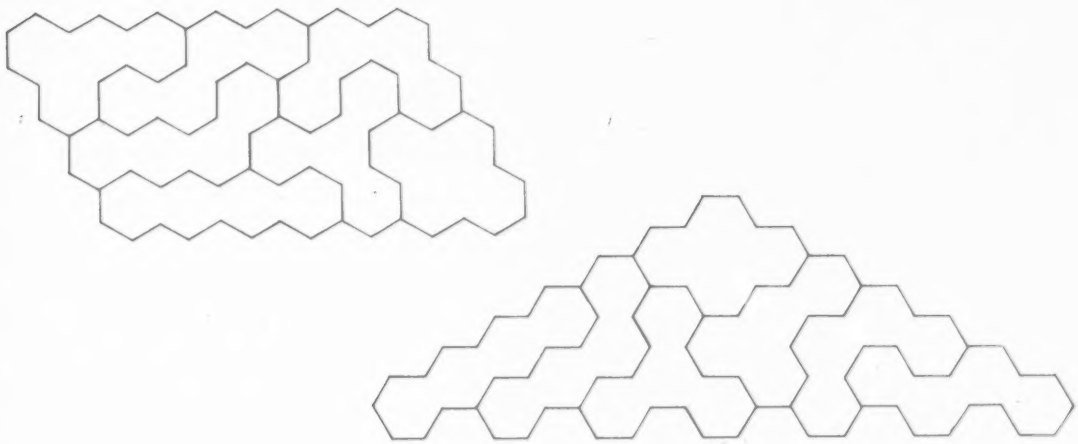
How many different shapes can you make on the grid with 3 hexagons? Make tessellation patterns using the shapes.



How many shapes can you make with 4 hexagons?

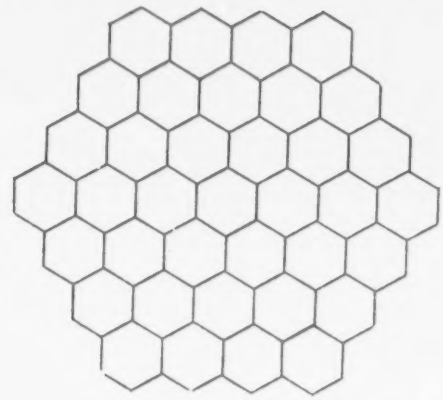


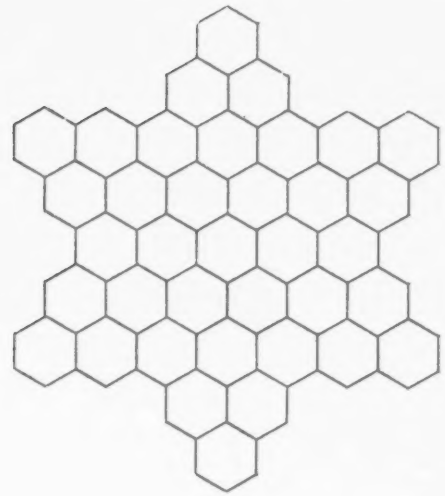
The seven shapes can be arranged to make other interesting shapes.
E.g.



If the shapes that can be made from 3 hexagons are combined with these made from 4 hexagons (a total of 10 shapes) then many more interesting designs are possible.

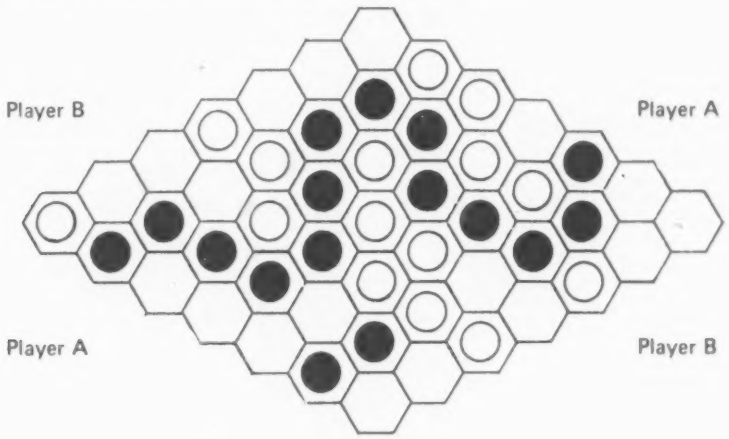
See if you can make these shapes:





GAME - HEX (2 players)

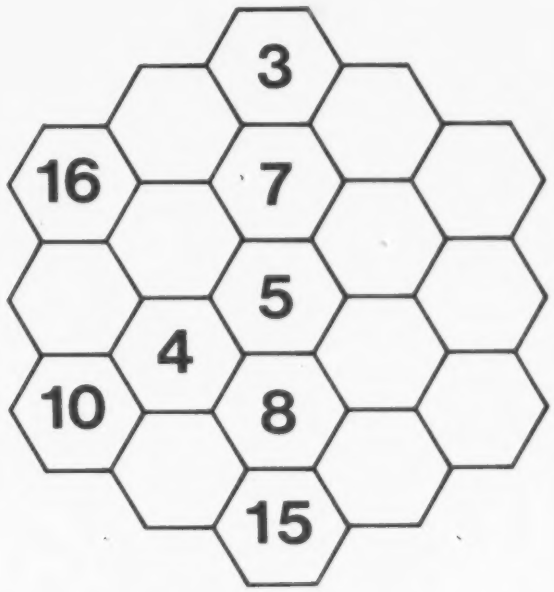
Materials needed: Playing area - outlined on hexagonal grid, 40 counters in 2 contrasting colours (centicubes are ideal).



Object: To be the first player to complete an unbroken chain of counters within the hexagons, from one side to the opposite side of the playing area.

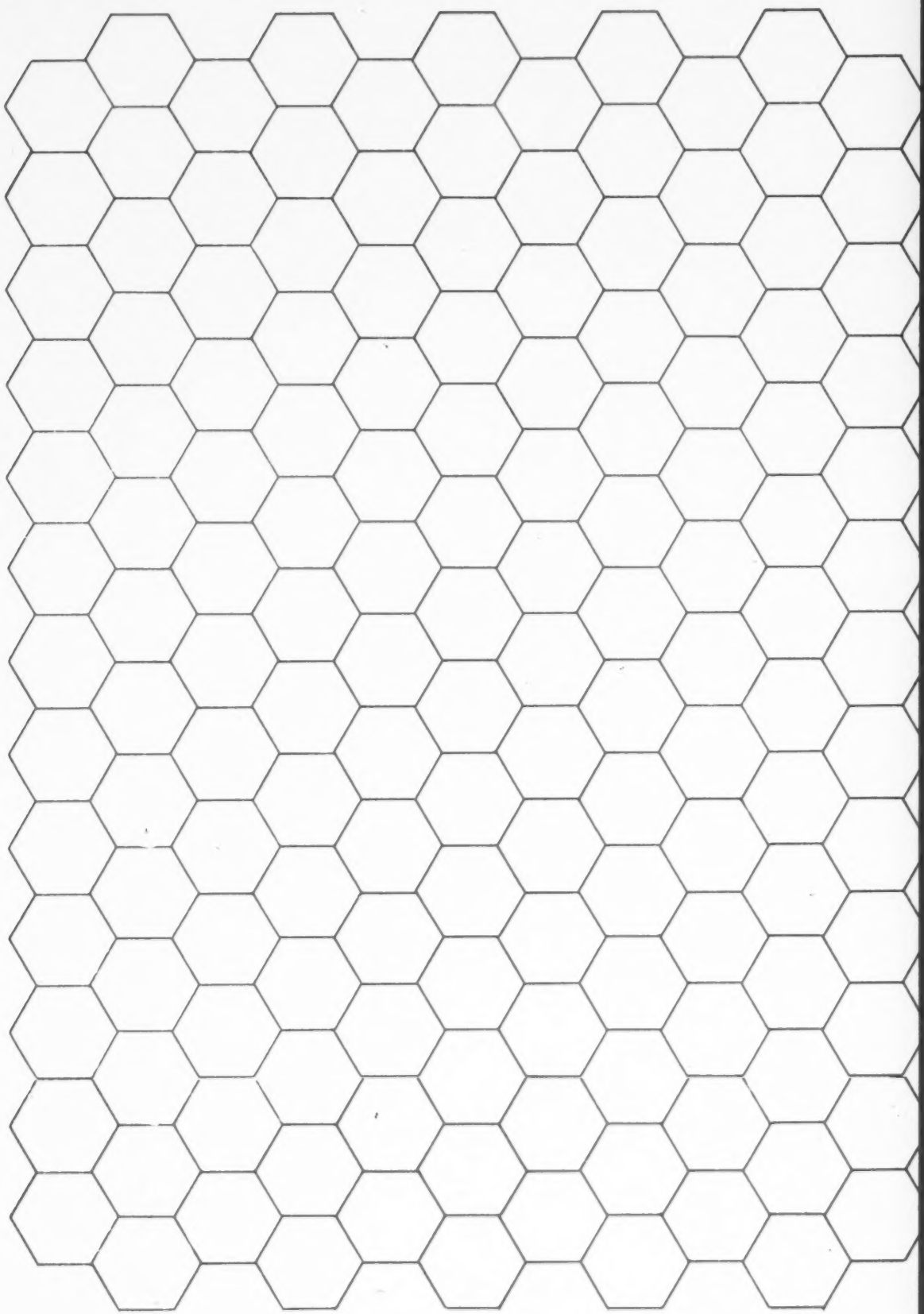
Rules: Players take turns to place a counter on the board, into any unoccupied hexagon.
The corner hexagons belong to either side and the winning chain may wander anywhere on the board.
The diagram shows the result of a game in which the black counters have won.

MAGIC HEXAGON

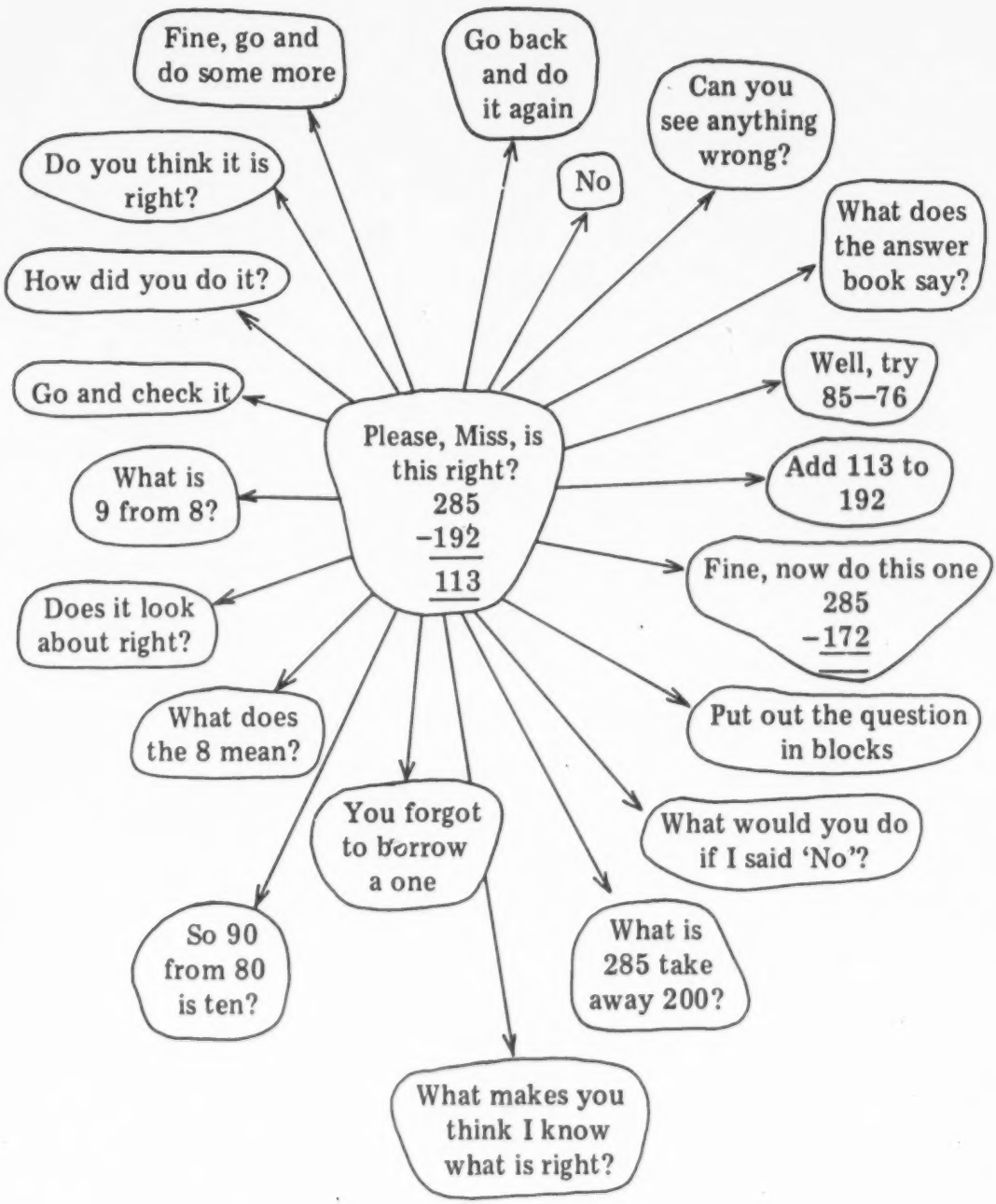


Complete the diagram so that the sum of the numbers in each vertical and diagonal set of hexagons is the same.

* * *



DECISIONS AND OPTIONS


Please, Miss, is this right?
285
−192
113
Fine, go and do some more
Go back and do it again
No
Can you see anything wrong?
Do you think it is right?
How did you do it?
What does the answer book say?
Go and check it
Well, try 85—76
What is 9 from 8?
Add 113 to 192
Fine, now do this one
285
−172
Does it look about right?
What does the 8 mean?
Put out the question in blocks
You forgot to borrow a one
What would you do if I said 'No'?
So 90 from 80 is ten?
What is 285 take away 200?
What makes you think I know what is right?

DOING CALCULATIONS, MAKING DECISIONS, PROBLEM SOLVING

Mathematics has traditionally been presented to children in such a way that very few, if any, decisions have to be made. In fact, some teachers go to great lengths in their lesson preparation to eliminate the need for decision making by them or their pupils during class time. Children practise calculations over and over again without having to give any thought as to what the figures are, where they came from and what the answer represents. No need to think, ask questions or make decisions - just follow the routines for which they have been programmed.

Everyday, children are presented with calculations in textbooks or on worksheets and all they are required to do is fill in the answers.

E.g.

$$\begin{array}{r} 451 \\ 23 \\ +1384 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 3204 \\ -\ 1768 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 123 \\ \times\ 64 \\ \hline 0 \end{array} \qquad 6\overline{)724}$$

No decisions have to be made about 'setting out' and because of this important considerations are missed.

To set out the addition problem like this a working knowledge of place value is required.

To set out the subtraction problem a decision has to be made about the relative 'size' of the two numbers.

Why is the multiplication problem set out the way it is?
Why not $\begin{array}{r} 64 \\ \times\ 123 \\ \hline 0 \end{array}$ and what is the 0 for?

When I asked a child to do the division problem using a calculator he obtained the answer 0.0082872 and said the calculator was faulty! He had interpreted the problem as $6 \div 724$.

Because all the calculations are divorced from any real situation it is not felt necessary to think about the reasonableness of the answers.

Examine the calculations below:

```
  1002        25
- 995       x 16
------      ----

5)515

  hr    min
   4     55
       x  6
  ----------
```

Do they need to be set out this way?
Do they need to be set out at all?

Children should be taught a whole range of strategies and short methods. When calculations have to be done many children should be in a position where they can make decisions about the figures and maybe choose to do the arithmetic mentally, with a pencil and paper - or sometimes with a calculator. Too often we see calculations like this:

```
   325          638.5
 x 100       .10)6385
   000
  0000
 32500
 32500
```

which show that children have become slaves to the formal algorithm.

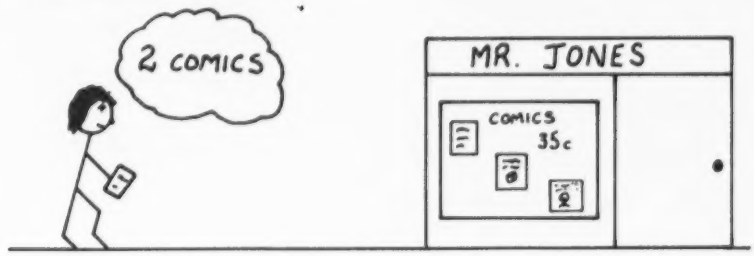
Number skills are important. They do need to be practised but the time spent on doing isolated, meaningless calculations should be decreased. A much greater emphasis needs to be placed on solving worded ('story') problems and using number skills in practical situations.

Except for the very intelligent, most children brought up on a diet of pure calculations find worded problems extremely difficult to solve. Even though teachers select the problems carefully, ensuring that the calculation required and the reading and comprehension levels are well within the ability range of the child, they still find it difficult. There are many reasons for this and one of the main ones is that problem solving

skills have to be taught.

Children have to be able to visualise what the problem is about. To help with this in lower grades children should 'act out' the story problem and then progress to drawing the situation. They must be encouraged to talk about the problem and make decisions about the information it contains.

E.g.
John had $1-00. He went to Mr Jone's shop to buy 2 comics. If the comics were 35¢ each how much money did he have left?



What is important?
Is it important that:

the boy's name was John?
he had $1-00?
the shopkeeper's name was Mr Jones?
he bought comic books?
the comics were 35¢ each?
he bought 2 comics?

To help children focus on different aspects of problem solving it is necessary to vary the structure of the problem and to ask questions about the information which has been given. Examples of four different types are given below:

* How long will my medicine last?
  Select the relevant pieces of information from the list below:
  a) I have to take the medicine 3 times a day.
  b) I have a sore throat.
  c) The bottle holds 250 ml.
  d) I take 10 ml each time.
  e) The medicine cost $6-50.

* What information is needed in order to be able to determine:
  a) your age in days;
  b) the total distance a batsman ran during his innings?

* Describe a situation (write a 'story problem') which would give rise to the calculation:
  a) \$10-00 - \$3-51 = \$6-49
  b) 15 + 3 x 2 = 21
  c) 6 ÷ ½ = 24

* Carole and Jennifer go to a shop. Carole has 65¢ and Jennifer has 90¢. They want to buy a book that costs \$1-20.
  Make up some questions to ask about this information.
  E.g. a) How much money do the girls have altogether?
  b) How much more money does Carole need in order to have the same as Jennifer?
  c) Carole wants to buy the book herself. How much more money does she need?
  d) The girls decide to buy the book between them and pay half each. How much money do they each have left?
  e) How much would 2 books cost?

Story problems do not usually contain the words add, subtract, multiply or divide. Children need practise at identifying the word or words in a problem which imply the operation which has to be carried out. E.g. for subtraction, it might be: lost, died, difference between, spent, how much more, take, gave away ...

Children should also be encouraged to write the answers to problems as complete sentences.

There is no method or set of strategies which, when applied, will guarantee a correct answer to every problem. However, the following check-list will prove useful in getting children to order their thoughts about a particular problem.

a) Identify the problem.
b) Decide what pieces of information are relevant.

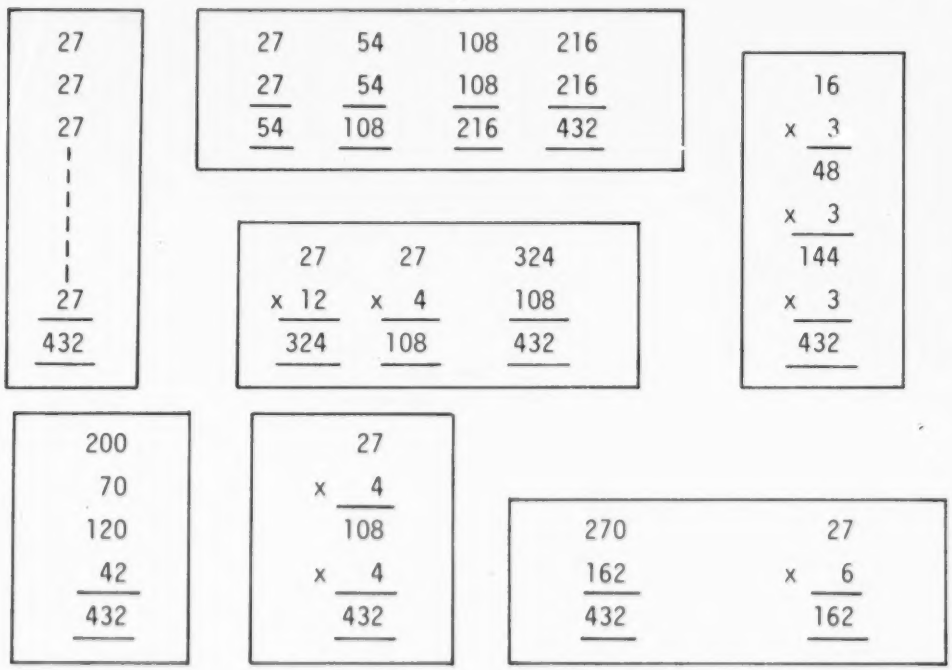
c) Make decisions on what to do with the relevant information:
   i) which numbers are needed first;
   ii) which arithmetical operation is appropriate.
d) Do the arithmetic - maybe choose a calculator.
e) Interpret the answer.
f) Check the reasonableness of the answer.

Whenever a new piece of mathematics is to be introduced to a class, it is often quite useful to present it in the form of a problem and allow the children to find a solution from the mathematics that they know.

E.g. Suppose a group of children 'know their tables' are 'friendly with numbers' and the teacher wants them to progress to multiplication by 2-digit numbers.

The problem can be posed: 'Find 16 x 27'.

The children may use a variety of methods in order to get the answer:



The various strategies, successful or otherwise, can then be discussed; the problem can be extended, patterns found and rules formulated. Children should be encouraged to find the most efficient way of recording. It is

extremely important that children are allowed this opportunity to explore a situation first, before any rule or algorithm is developed.

Children should also have the opportunity from time to time to take part in what I call 'real problem solving'. This involves either the whole class or groups of 3 or 4 children working together, talking about mathematics, writing about mathematics and making decisions, in an effort to solve the task which they have been given. The tasks may require a knowledge of some aspects of measurement or spatial work and involve calculations. In the upper part of the primary school calculators should be provided so that attention can be focussed on the problem and its solution rather than be diverted by complex arithmetic.

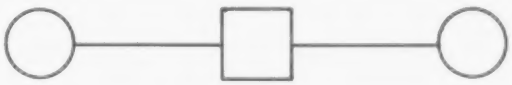
Below are some problems you might like to try, or adapt for use with your class.

* Find all the numbers less than 500 which have exactly 9 factors.
* In how many ways can you give change for 20¢?
* How high is the school flagpole?
* How much water is wasted each day by a dripping tap?
* Draw a circle on the playground which has a circumference of exactly 9 metres.
* Suppose you want to get as far away as possible. How far can you get in 6 hours?
* How long would it take to recite all the number names from one to a million?
* Construct a piece of apparatus which will measure time intervals of 30 seconds?
* In how many ways can you measure a tennis ball?
* Find a number which when multiplied by itself is equal to 8. (A calculator is needed for this question - without a square root button.)
* What is the largest area you can enclose with a piece of string of length 36cm?
* Construct accurately all the triangles which have a perimeter of 24cm. The length of each side must be a whole number of centimetres.

* * *

ARITHMAGONS

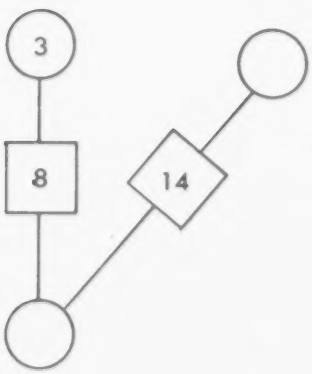
RULE: The number in the square is always the sum of the numbers in the circles on either side of it.

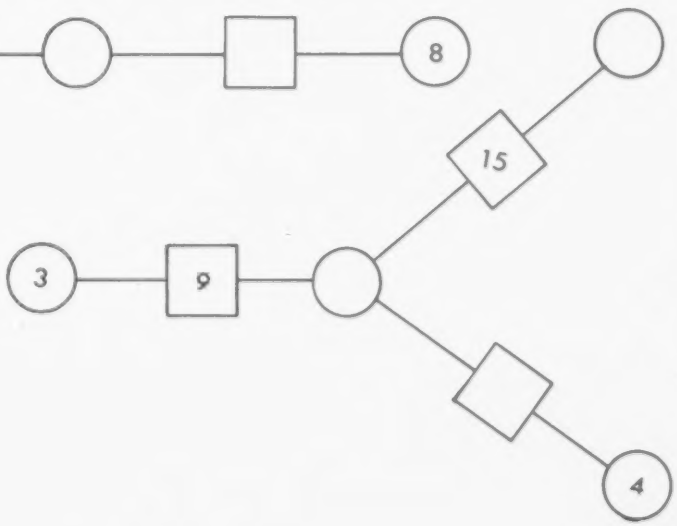


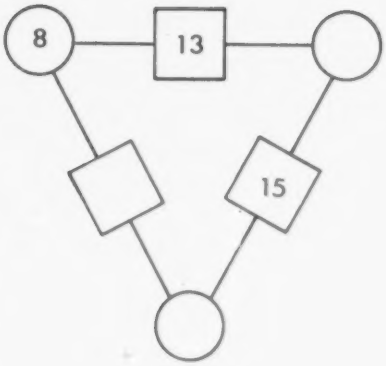
This very simple starting point can be adapted for use in any grade - including infants. It provides practice in addition and subtraction and is a good problem solving activity. Some children would need large diagrams and have access to number cards and counters.

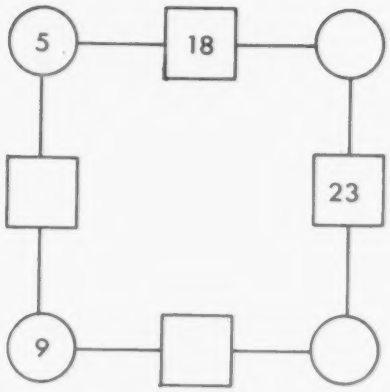
A few of the possible variations are shown below:

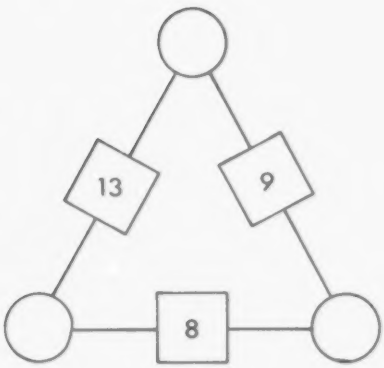
Complete the diagrams.





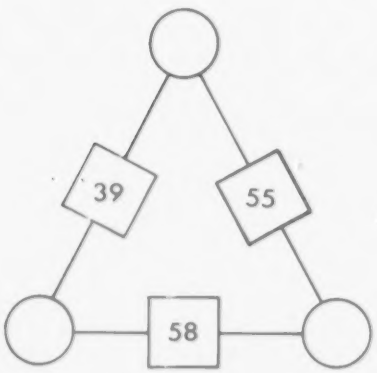






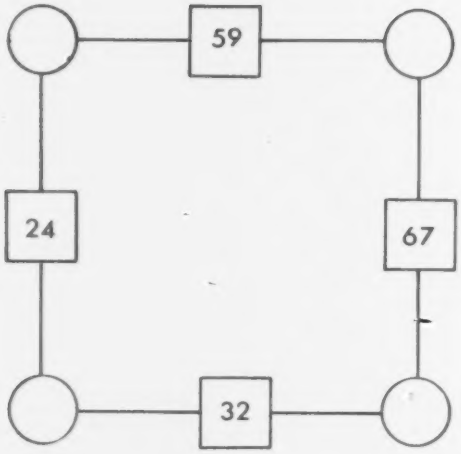
This is a more difficult problem but a trial and error approach will soon provide a solution.

If the numbers are larger then a better method is required.



Investigate problems of this type with your class. Encourage the children to make up their own examples; look for patterns and formulate rules. See if they can discover a rule whereby any circle number can be determined easily by applying the rule to the numbers in the squares.

Now try it with squares.



Start again with a new rule, for example:
'The number in the square is always the product of the numbers in the circles on either side of it'.

* * *

If you had \$1 000 000 and you gave away \$10 every hour, how long would it take to give away all your money?

*

Find a fraction which is greater than $\frac{2}{3}$ but less than $\frac{3}{4}$.

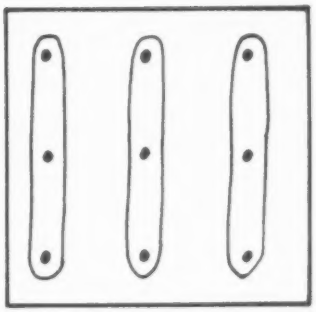
*

A lake resort owner rented a cabin for 14 days on the condition that he would receive \$40 a day for every day that it did not rain and \$10 a day for every day it did rain. At the end of the 2 weeks the resort owner received \$380.

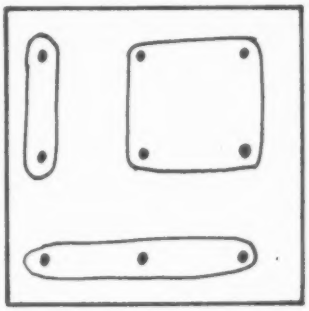
How many days did it rain?

*

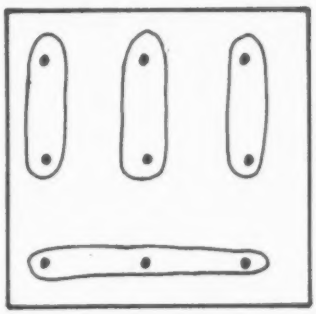
'THE STORY OF 9' USING A NAILBOARD



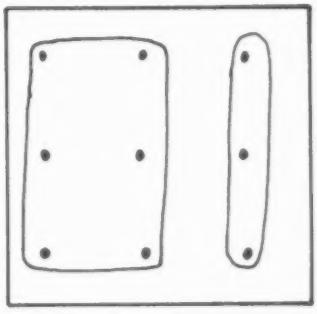
3 lots of 3



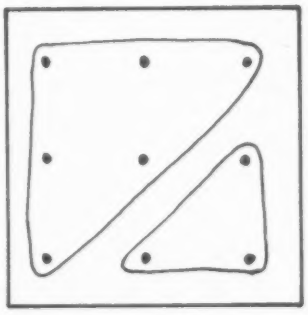
2 + 3 + 4



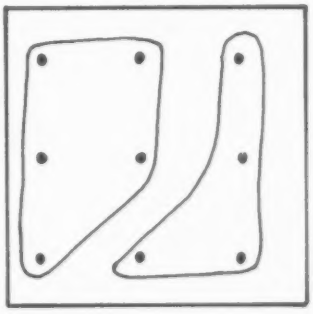
3 x 2 + 3



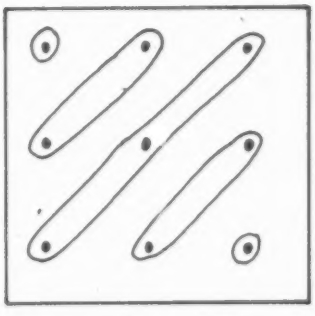
6 + 3



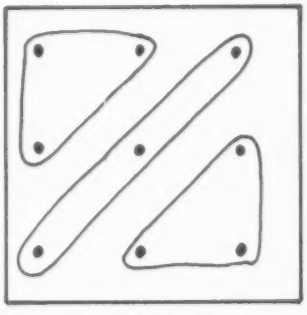
6 + 3



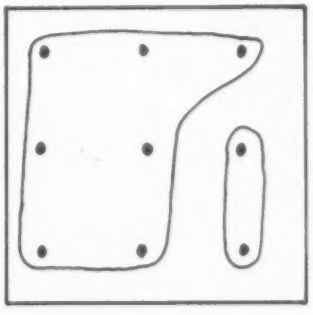
5 + 4



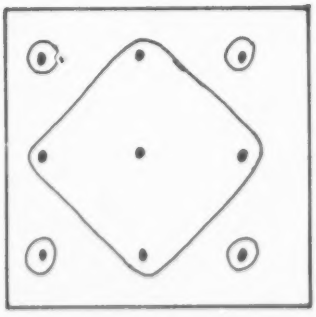
1 + 2 + 3 + 2 + 1



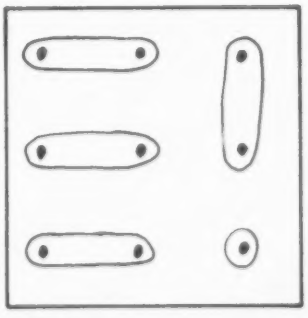
3 + 3 + 3



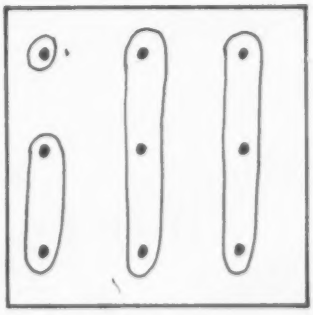
7 + 2



5 + 1 + 1 + 1 + 1



4 x 2 + 1



1 + 2 + 3 + 3

1, 2, 3, 4, 5, 6, 7, 8, 9

All the problems below involve the numbers from 1 to 9. They can be set as challenges to individuals, small groups or the whole class. Children should be encouraged to persist and think carefully about the situations rather than be allowed to 'give up' if their first few trial and error attempts fail.

A set of 9 cards (numbered 1 - 9), approximately 3cm square, are required for each problem.

a) Arrange the cards in such a way that you can easily find the sum of the numbers from 1 to 9.

1, 9
2, 8
3, 7
4, 6
5 } = 4 x 10 + 5 = 45

2, 9
3, 8
4, 7
5, 6
1 } = 4 x 11 + 1 = 45

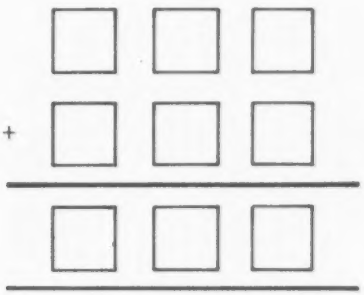
8, 1
7, 2
6, 3
5, 4
9 } = 5 x 9 = 45

9, 6
7, 8
1, 2, 3, 4, 5 } = 3 x 15 = 45

b) Make up number problems so that the complete set of 9 numbers is used for the question and answer. For example:

$$\begin{array}{r} 12 \\ 3 \\ 6 \\ 7 \\ 8 \\ +\ 9 \\ \hline 45 \end{array} \qquad \begin{array}{r} 467 \\ +\ 352 \\ \hline 819 \end{array} \qquad \begin{array}{r} 927 \\ -\ 586 \\ \hline 341 \end{array} \qquad \begin{array}{r} 1738 \\ \times\ 4 \\ \hline 6952 \end{array} \qquad 4\overline{)7852}\ 1963$$

Alternatively children can be presented with the question format, for example:



and asked to place their cards in the spaces in such a way as to give a correct result.

A challenging variation is to make up sets of equations.

E.g.

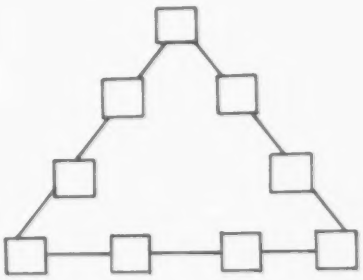
$1 + 2 \times 3 + 4 + 5 - 6 + 7 - 8 = 9$

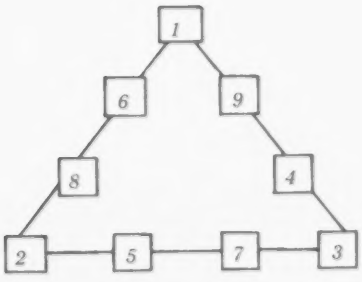
$3 \times 2 = 6$
$4 + 5 = 9$
$8 - 7 = 1$

$\frac{1}{2} \times 6 = 3$
$57 - 48 = 9$

$12 \times 7 = 84$
$3 \times 5 - 6 = 9$

c) Place the cards so that the sum of the numbers on each side of the triangle is the same.

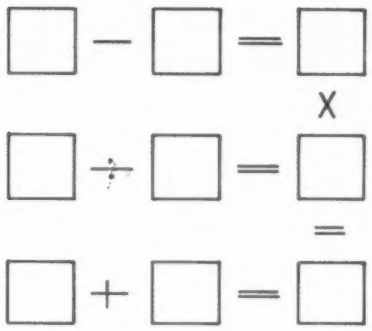




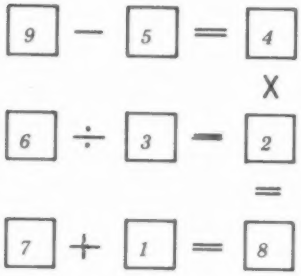
The sum of the numbers on each side is 17.

It is possible to find triangles in which the sum of the numbers is 19, 20, 21 and 23.

d) Place the numbers 1 to 9 in the spaces so that each horizontal and vertical set of 3 numbers forms a true equation.

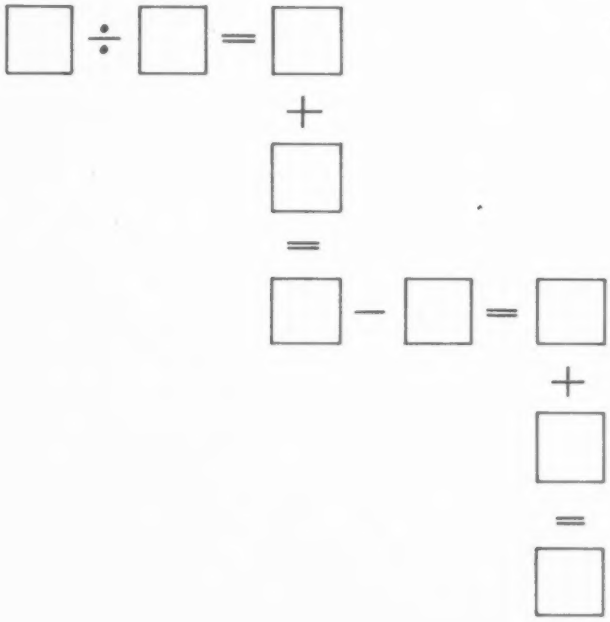


The answer is:



Now try the next one.

Encourage the children to make up equation puzzles of their own.

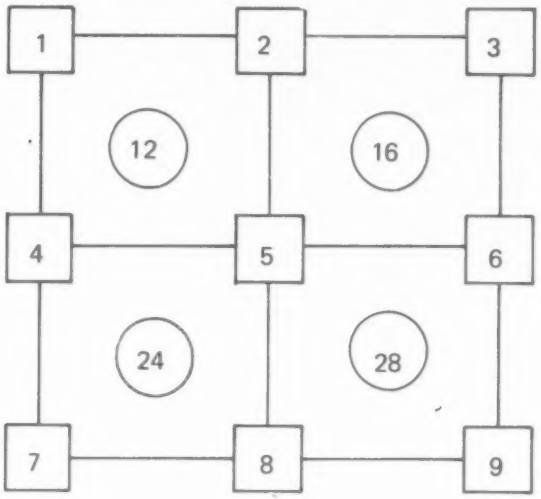


e) Keeping the numbers in order, insert mathematical signs as necessary to make a total of 100.

There are many ways to do this. One solution is:

$$12 + 3 + 4 + 5 - 6 - 7 + 89$$

f) Place the cards as shown below.



The number in each circle is the sum of the four numbers in the square which surrounds it.

Arrange the cards so that the sum of the numbers in each square is the same.

Nine different totals are possible ranging from 16 to 24. For example:

9 — 4 — 8 | (16) | (16) | 2 — 1 — 3 | (16) | (16) | 6 — 7 — 5	2 — 7 — 3 | (19) | (19) | 9 — 1 — 8 | (19) | (19) | 5 — 4 — 6	1 — 6 — 2 | (24) | (24) | 8 — 9 — 7 | (24) | (24) | 4 — 3 — 5

g) i) Choose 3 of the numbers e.g. 2, 5, 8.

ii) Form all possible 2-digit numbers.

25, 28, 52, 58, 82, 85

(there will always be six numbers).

iii) Find the sum of the numbers in (i) 2 + 5 + 8 = 15

iv) Find the sum of the numbers in (ii)

25 + 28 + 52 + 58 + 82 + 85 = 330

v) Divide (iv) by (iii). 330 ÷ 15 = 22

The answer will always be 22.

h) A game for 2 players.

Place the 9 cards face up on the table.
Players select cards in turn and win the game if they have in their possession a set of 3 cards which have a total of 15.
E.g.

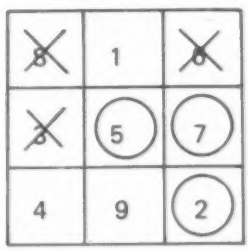
Player A (goes first)	Player B
7	6
5	3
2	8
1	4

B wins (3 + 8 + 4 = 15)

This game can be thought of as playing O's and X's on a 3 x 3 magic square which has rows, columns and diagonals totalling 15.

Consider the game above in which A's move is denoted by O and B's move by X.

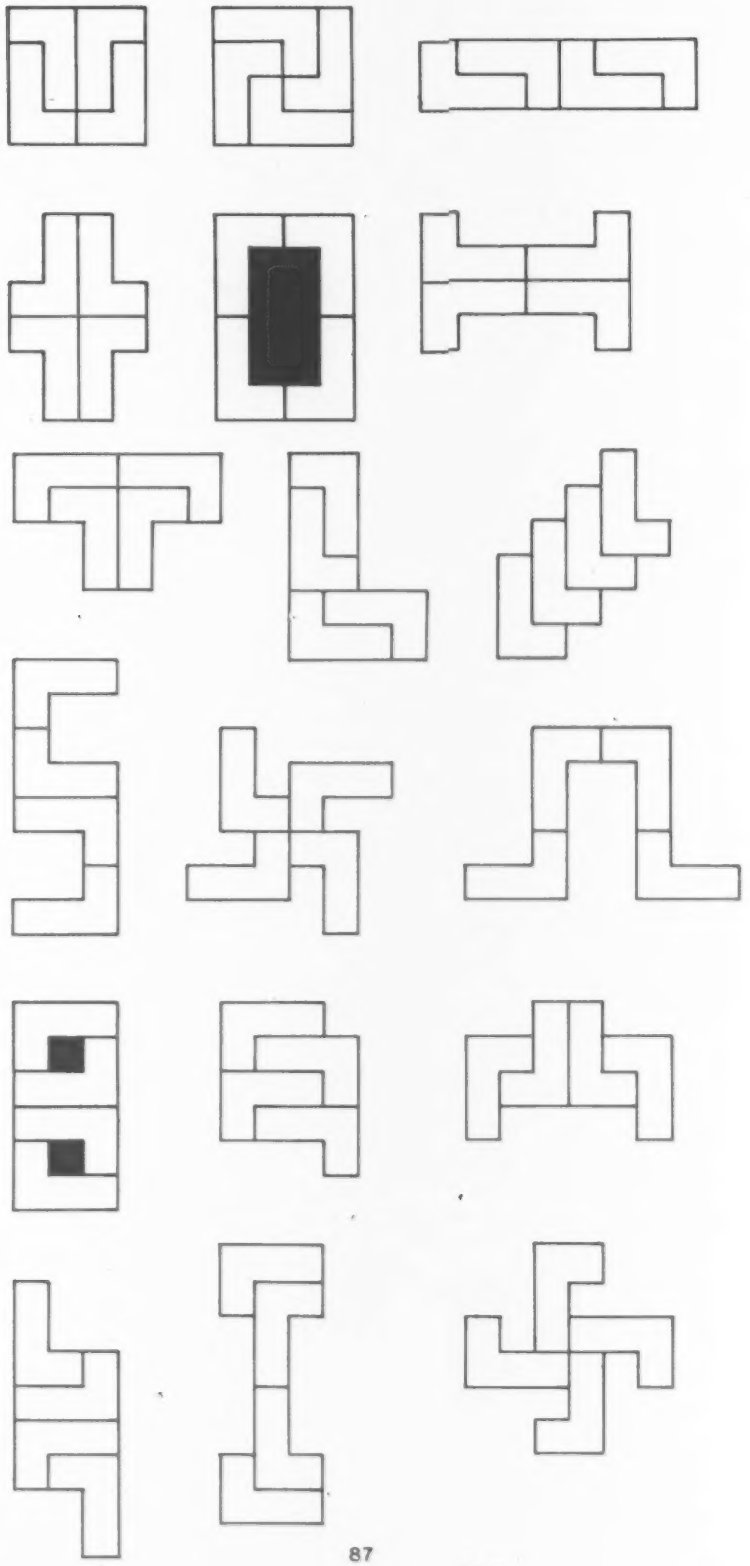
After the first three moves the situation will be as shown in the diagram.



There are now only three numbers left to choose from and whichever one A takes player B cannot lose.

* * *

WITH FOUR L's



87

ACTIVITIES USING THE 1-100 NUMBER SQUARE

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

How many numbers are there in each row?

Where are all the numbers that have a 3 in the units position?

Where are all the numbers that have a 6 in the tens position?

What can be said about the numbers in the 4th row?

What can be said about the numbers in the 8th column?

If you extend the number square downwards and continue the numbering beyond 100, in which column will you find: 103, 111, 204, 795, 1080?

How many tens in 50, 70?

Find the numbers with 2 digits the same. Where are they?

Add 10 to any number.

Subtract 10 from any number.

Add 9 to any number - (add ten and move back one).

Subtract 9 from any number - (subtract 10 and move on one).

Add 20, 30 etc. to any number.

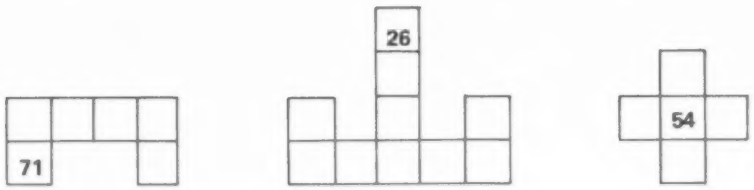
Add any single digit number and investigate the patterns:

E.g. 4 + 5 = 9 2 + 3 = 5 8 + 7 = 15

14 + 5 = 19 12 + 3 = 15 18 + 7 = 25

24 + 5 = 29 22 + 3 = 25 28 + 7 = 35

Fill in the blanks in these extracts from the square.



Shade in the multiples of 2, 3, 4, ... 12 and investigate the patterns formed.

E.g.

1	2	3	4	~~5~~	6	7	8	9	~~10~~
11	12	13	14	~~15~~	16	17	18	19	~~20~~
21	22	23	24	~~25~~	26	27	28	29	~~30~~
31	32	33	34	~~35~~	36	37	38	39	~~40~~
41	42	43	44	~~45~~	46	47	48	49	~~50~~
51	52	53	54	~~55~~	56	57	58	59	~~60~~
61	62	63	64	~~65~~	66	67	68	69	~~70~~
71	72	73	74	~~75~~	76	77	78	79	~~80~~
81	82	83	84	~~85~~	86	87	88	89	~~90~~
91	92	93	94	~~95~~	96	97	98	99	~~100~~

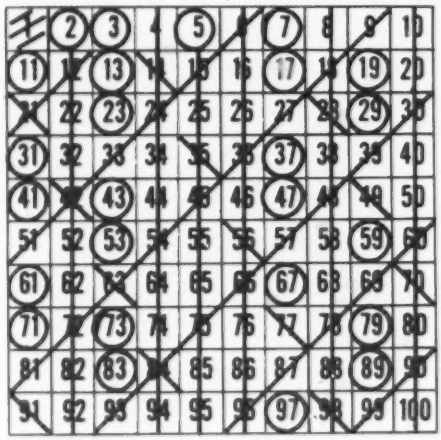
Multiples of 5

1	2	3	4	5	6	7	8	~~9~~	10
11	12	13	14	15	16	17	~~18~~	19	20
21	22	23	24	25	26	~~27~~	28	29	30
31	32	33	34	35	~~36~~	37	38	39	40
41	42	43	44	~~45~~	46	47	48	49	50
51	52	53	~~54~~	55	56	57	58	59	60
61	62	~~63~~	64	65	66	67	68	69	70
71	~~72~~	73	74	75	76	77	78	79	80
~~81~~	82	83	84	85	86	87	88	89	~~90~~
91	92	93	94	95	96	97	98	~~99~~	100

Multiples of 9

This should lead to a discussion on odd and even numbers, common multiples and rules for divisibility.

Cross out 1, and then all the multiples of 2 except 2 itself; all the multiples of 3 except 3 itself; and do the same for 5 and 7. We are left with all the prime numbers less than 100. We do not need to go beyond the multiples of 7. Why not?



This crossing-out method of selecting prime numbers is usually called Eratosthenes' Sieve. Mathematicians in general choose to regard 1 as neither prime nor composite.

Look for patterns in the arrangement of the numbers in the square and find an easy way for calculating the sum of the numbers from 1 to 100.
E.g. By adding rows:

$$1 + 2 + 3 + \cdots\cdots + 10 = 55$$
$$11 + 12 + 13 + \cdots\cdots + 20 = 55 + 100$$
$$21 + 22 + 23 + \cdots\cdots + 30 = 55 + 200$$
$$\cdots\cdots\cdots

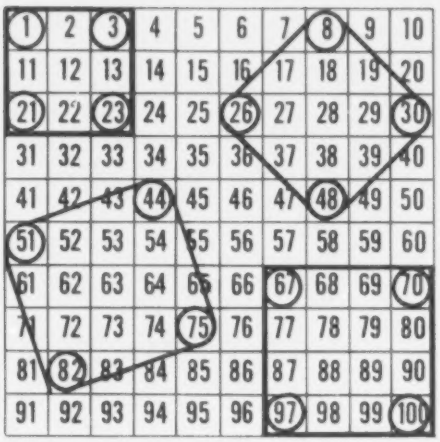
By adding in pairs:

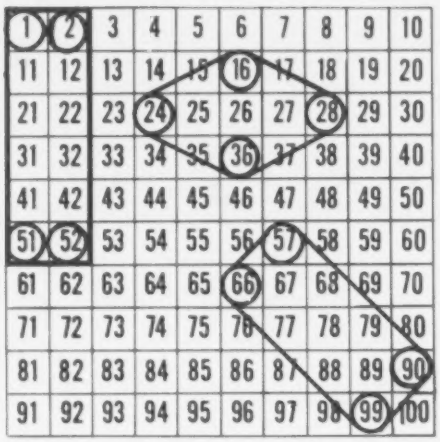
$$\begin{array}{r} 1 + 99 = 100 \\ 2 + 98 = 100 \\ 3 + 97 = 100 \\ \hline 49 + 51 = 100 \\ \hline = 4900 \\ + \ 100 \\ + \ 50 \\ \hline = 5050 \end{array}$$

OR

$$\begin{array}{r} 1 + 100 = 101 \\ 2 + 99 = 101 \\ 3 + 98 = 101 \\ \hline 50 + 51 = 101 \\ \hline = 5050 \end{array}$$

These numbers were left without a partner.

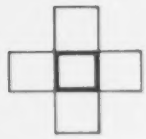




Outline any square on the grid.
What can be said about the numbers in opposite corners?
Does the same thing happen for all squares?
Does it work for rectangles?
Does it work for parallelograms?

If the numbers 34, 36, 64 are three of the corners of a parallelogram what are the possible positions for the fourth corner?

We need not limit ourselves to squares, rectangles and parallelograms.
Draw a cross anywhere on the square.
Add together all five numbers in the cross.
What is the connection between this sum and the middle number?
Is this connection true for all crosses?
How does the rule change when crosses are drawn with longer arms?



Draw shapes of your own design on the square. Can you find rules which enable you to calculate the sum of all the numbers contained within these shapes without actually adding them all together?

* * *

John collects lizards, beetles and worms.

Altogether in the collection there are 11 heads and 38 legs.
He has more beetles than worms.
How many of each creature does John have?

*

Replace the four letters in the addition problem by the four numbers 1, 9, 8, 3 - different numbers for different letters.

BAD + MAD + DAM =

What is the largest total you can find?

*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

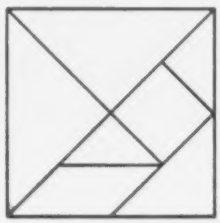
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

TANGRAMS

Quite often the tangram, together with many other similar puzzles, has been introduced into the primary and secondary classroom as a means of some light relief at the end of term. However, teachers are now beginning to appreciate the value of such puzzles as an integral part of the mathematics programme.

The strategies for solving dissection puzzles like the tangram involve consideration of shape, lengths of side, angles, area and symmetry of the individual pieces. At the same time children can learn in an informal way a great deal of elementary geometry, as well as important concepts such as the conservation of area.

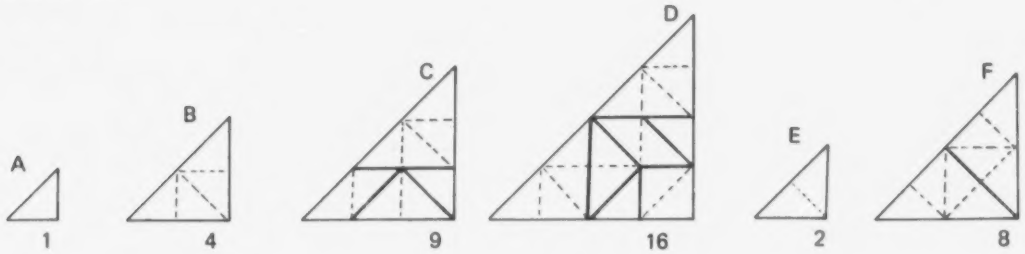
The Chinese tangram is probably one of the oldest and most popular dissection puzzles. It consists of 7 pieces cut from a square as shown in the diagram. Usually children work at problems which involve using all 7 of the pieces to make designs and representations of human figures, animals, etc.



One question that we frequently ask in mathematics is "How many?" and the work with the tangram is no exception.

How many different triangles, squares, ... can you make using some or all of the pieces?

Six different triangles can be made and these are all right-angled and isosceles.



If the small triangle is taken as the 'unit of area' then the area of the other triangles can easily be found and compared.

If children are asked to draw a shape 'twice as big' as a given shape, several different answers could result. 'Twice as big' normally means that each dimension of the original shape is to be doubled to produce the new shape.

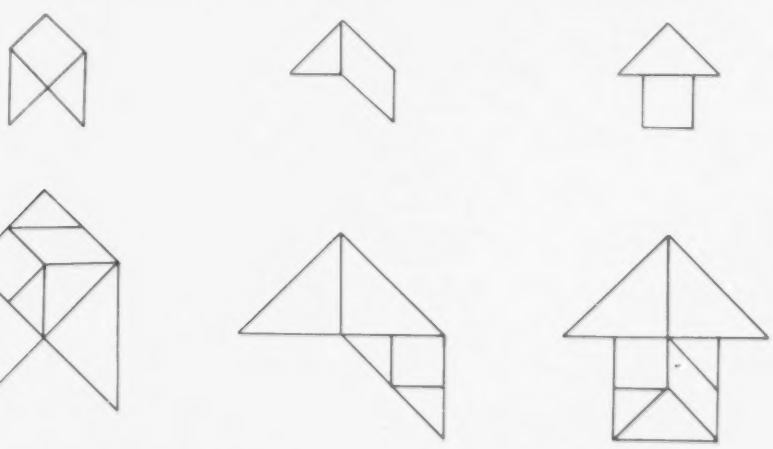
△ B is twice as big as △ A
△ D is twice as big as △ B
△ F is twice as big as △ E
△ C is three times as big as △ A
△ D is four times as big as △ A

Notice how the area changes as the triangle is enlarged. The sides of △ C are three times as long as those of △ A and the area of △ C is $9(3^2)$ times as big as △ A. The area changes by the square of the enlargement and this is true for all shapes.

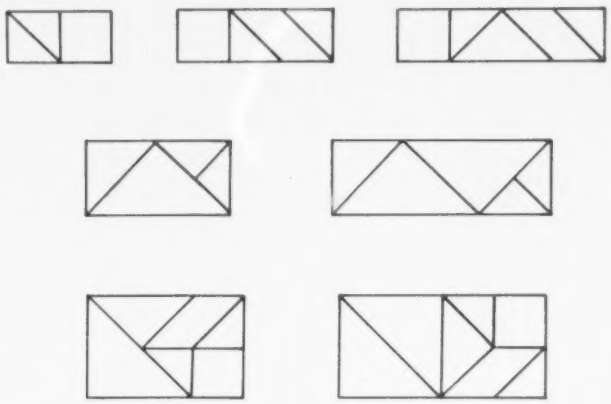
△ E is twice as big as △ A in <u>area</u>, the enlargement is, in fact, $\sqrt{2}$.

It is very worthwhile exercise to study enlargements with the other sets of shapes which can be made from the tangram pieces.

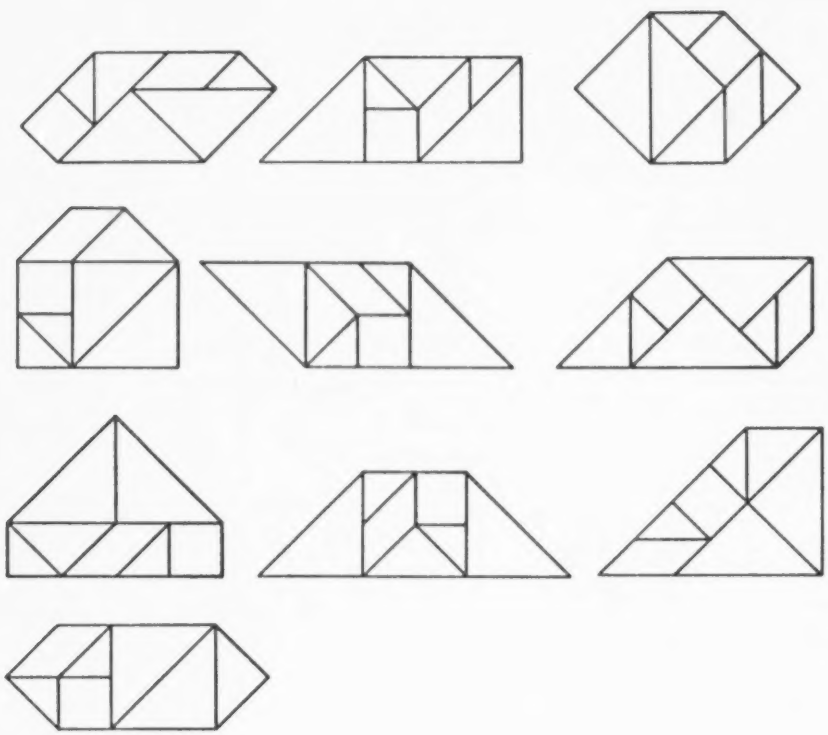
One way of doing this, is for the children to work in pairs. One person selects pieces from his tangram set and makes a shape with an area of not more than 4 unit triangles. The other person then has to arrange his pieces to make an exact enlargement x2 of the shape.
E.g.



It is possible to make 7 different rectangles and these are shown below. I leave it to you and your class to determine how many squares, parallelograms, trapeziums, pentagons and hexagons are possible.



If we restrict ourselves to using all 7 pieces, 13 convex polygons are possible. The square, triangle and rectangle are shown above, the remaining 10 are shown below.

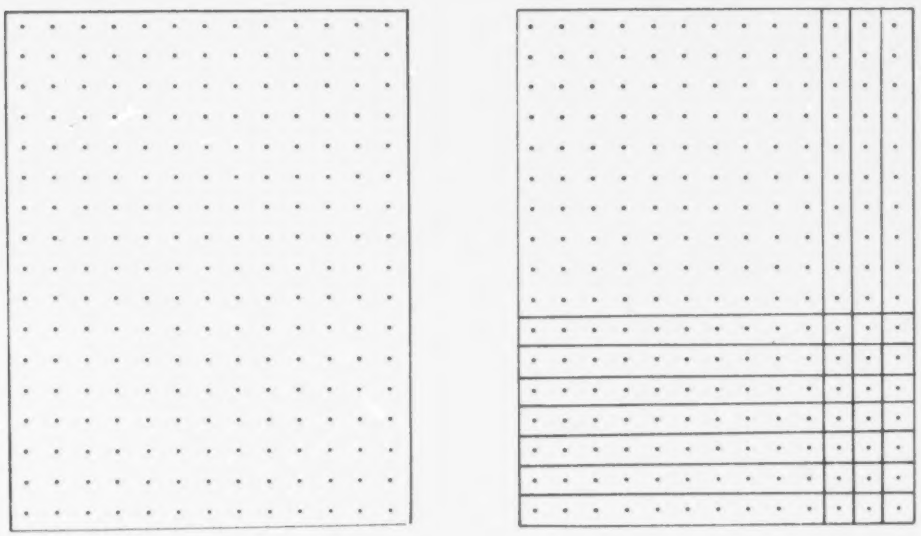


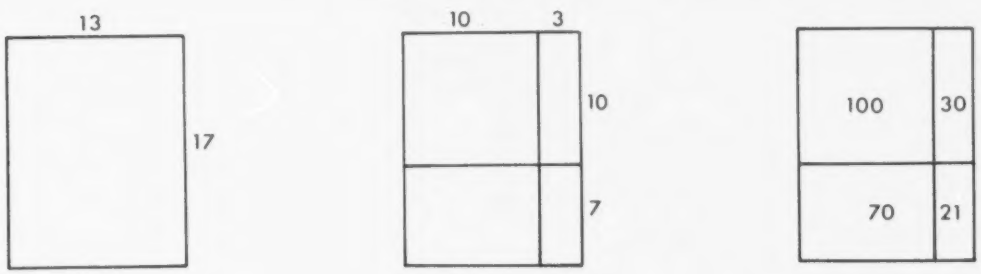
There are two books currently available containing hundreds of fascinating designs which can be made using the tangram pieces. (Both contain answers)

Ronald C. Read, *Tangrams . 330 Puzzles*, Dover Books, New York, 1965.
Joost Elffers, *Tangrams*, Penguin, Harmondsworth, 1976.
Dale Seymour, *Tangramath*, Creative Publications, Palo Alto, California, 1971.

* * *

HOW MANY DOTS?



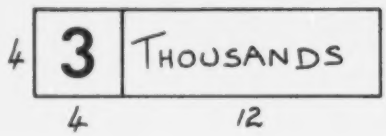


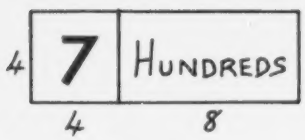
$13 \times 17 = 100 + 70 + 30 + 21$
$= 221$

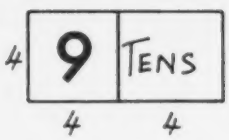
*

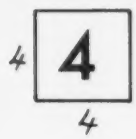
PLACE VALUE CARDS

A set of cards should be constructed similar to those below. Dimensions are in centimetres.



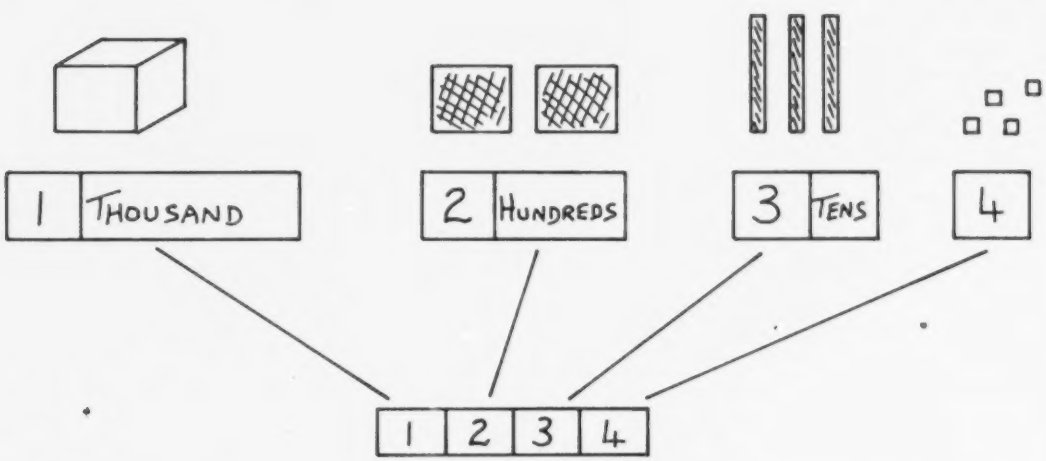




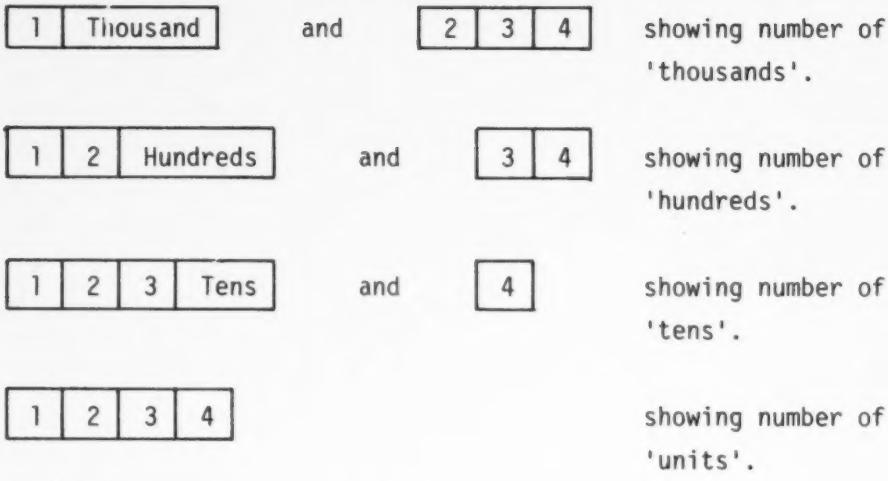


You will need nine 'thousands' cards showing the numerals 1→9 and ten of each of the other cards showing the numerals 0→9.

The cards can be used in conjunction with Dienes M.A.B. (base 10) as an aid to recording.



The cards can also be placed together in different ways to emphasise the structure of a particular number.



The ability to split a number up in this way is particularly useful for dealing with certain types of subtraction problems: those which have zeros on the 'top line'.

E.g.
```
  3 0 0 2
- 1 7 6 8
---------
---------
```

3002 can be thought of as 300 'tens' and 2. If 1 'ten' is taken (changed to units) then there will be 299 'tens' remaining.

```
  2 9 9,
  ~~~2
- 1 7 6 8
---------
---------
```

* * *

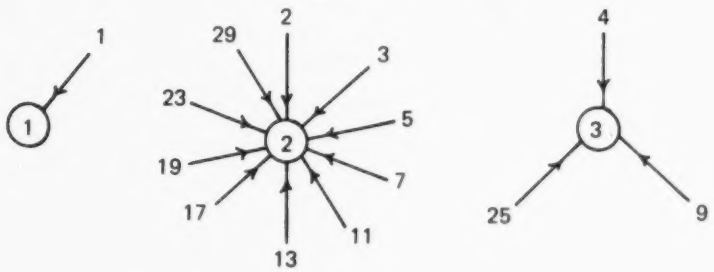
ACTIVITIES INVOLVING FACTORS

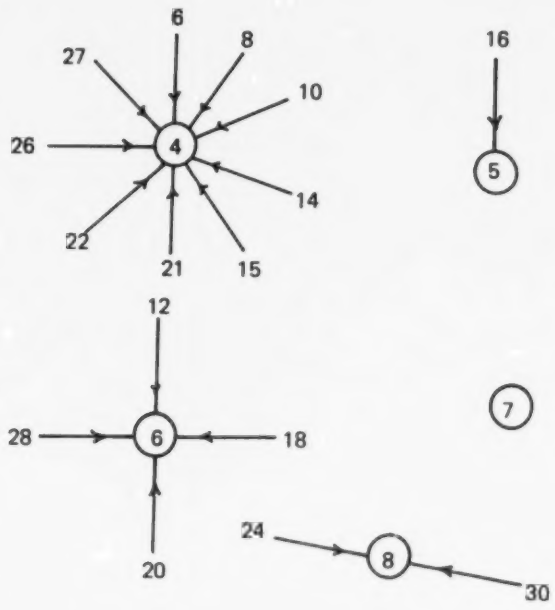
Factors are whole numbers which divide exactly into another whole number. E.g. 1, 2, 3, 6, 7, 14, 21, 42, are all factors of 42.

Children could be asked to construct a table similar to that below and complete it to show all the factors of the numbers from 1 to 30.

NUMBER	FACTORS	NUMBER OF FACTORS
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
⋮	⋮	⋮
30	1, 2, 3, 5, 6, 10, 15, 30	8

If all the numbers (1 - 30) which have the same number of factors are put together using mapping (spider) diagrams then some interesting patterns can be observed and some challenging questions asked.





What name is given to the numbers which have only 2 factors? (prime)
What name is given to numbers which have an odd number of factors? (square)
Can you find a number with 7 factors; 9 factors?
Which number less than 100 has the greatest number of factors?

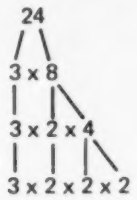
A rectangle has an area of 36 m^2. Each side is a whole number of metres long. What are the possibilities?

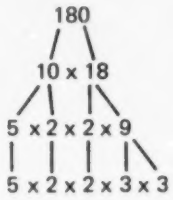
A rectangular box has a volume of 64 cm^3. Each side length is an exact number of centimetres. What are the possibilities?

Two numbers have a product of 84 and a sum of 20. What are the two numbers?

If the sum of the factors of a number - not counting the number itself - is equal to the number, then that number is called PERFECT.
The first perfect number is 6. (1 + 2 + 3 = 6)
What is the second perfect number?

All numbers can be expressed as the product of their prime factors. An easy way of doing this is to draw 'factor trees'.
E.g.





This provides a good opportunity to introduce indices as a shorthand way of writing the answers.

$$24 = 3 \times 2^3 \qquad 180 = 2^2 \times 3^2 \times 5$$

There are many ways to express 10 as the sum of two or more numbers:
E.g. $10 = 8 + 2$
$10 = 5 + 3 + 2$
$10 = 6 + 2 + 1 + 1$

If these numbers are multiplied the products are different:

$8 \times 2 = 16$
$5 \times 3 \times 2 = 30$
$6 \times 2 \times 1 \times 1 = 12$

a) Find the <u>largest</u> product of the numbers which add up to 10.
b) Find the largest product of the numbers which add up to 4, 5, 6, ... 12.
c) Can you find an easy way to obtain the largest product?
d) Find the largest product of the numbers which add up to 14, 15 and 16.

THE FACTOR GAME (2 players)

Materials Required: 20 cards numbered 2→21.

1. Set the cards out in a 4 x 5 rectangle as shown below:

2	3	4	5	6
7	8	9	10	11
12	13	14	15	16
17	18	19	20	21

2. Players take turns at selecting a number.
3. After a player has taken a number his opponent then takes all the factors of that number, which have not previously been taken.
   E.g. Player A starts and takes 21.
   Player B takes 3 and 7 (factors of 21) and then selects his number.
4. Play continues until all cards have been taken.
5. The winner is the player with the greatest total.

* * *

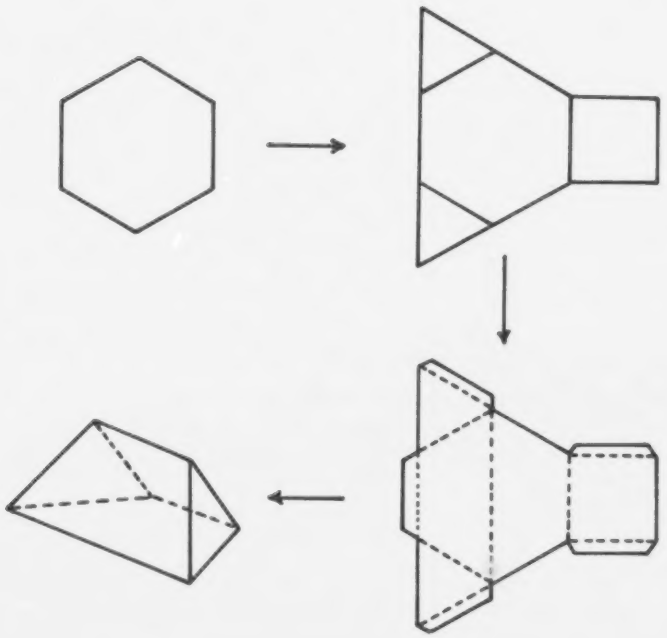
a) The sum of two numbers is 14, and their product is 24. What are the two numbers?

b) The sum of two numbers is 14, and their product is 27.84. What are the two numbers?

(You may use a calculator).

*

TETRAHEDRON PUZZLE



- Construct a regular hexagon.
- Construct a square and two equilateral triangles as shown in the diagram.
- Add joining flaps.
- Fold along the dotted lines.
- Glue and put together.
- Make 2 of these solids and fit them together to make a tetrahedron (triangular pyramid).

* * *

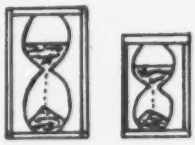
EGG PROBLEMS FOR EASTER

1. Find out from the local supermarket how eggs are graded and packaged. Find the cost of the various grades and determine which eggs provide the best 'value for money'.

2. If 3 chickens lay 12 eggs in 4 days, how many eggs will 5 chickens lay in 6 days? (Assume that they all lay at the same rate.)

3. If it takes 3 minutes to boil an egg, how long would it take to boil 5 eggs?

4. I have two egg timers. One times for 3 minutes and the other for 5 minutes. How could I use the timers to time for
   a) 8 minutes
   b) 2 minutes
   c) 1 minute ?



5. Harold Witcomb and Gerald Harding shelled 1 050 dozen hard-boiled eggs in 7¼ hours at Bowyers factory in Trowbridge, Wiltshire, on 23 April 1971.

   If both men worked at the same rate how long did it take one man to shell one egg?
   How long do you take to shell a hard-boiled egg?

6. In a basket are 27 eggs. The eggs are identical except for one which is heavier than the others. Describe how you could find the 'heavy egg' by using a balance - only 3 times.

7. The longest recorded distance for throwing a fresh hen's egg without breaking is 106.68m on the 58th exchange between William Cole and Jonathan Heller in Central Park, New York, USA on 17 March 1979.

   Measure out this distance on the school field (if its long enough). Hold a tennis ball throwing and catching competition.

8. How big was the largest Easter Egg ever made? Look in the Guinness Book of Records.

9. Eggs are sold in 1 dozen and ½ dozen boxes.
   1 dozen eggs cost $1-75.
   Mrs Jones has $5-00.

Make up 3 questions to ask about this information.

10. Large Easter Eggs are made from 300g of chocolate and cost $3-00 each.
Small Easter Eggs are made from 100g of chocolate and cost $1-20 each.
John has $6-00.
Mary has $5-00.
Make up 5 questions to ask about this information.

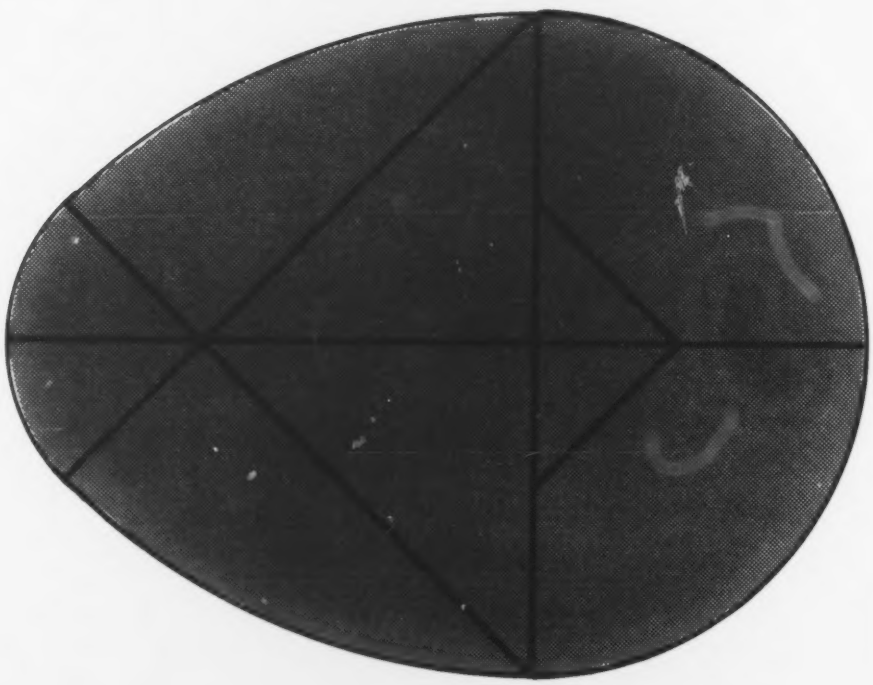
11. Find a digit to replace each letter in the addition problems below.
Each letter stands for only one digit.

a)
```
 H E N S
   L A Y
 -------
 E G G S
 -------
```

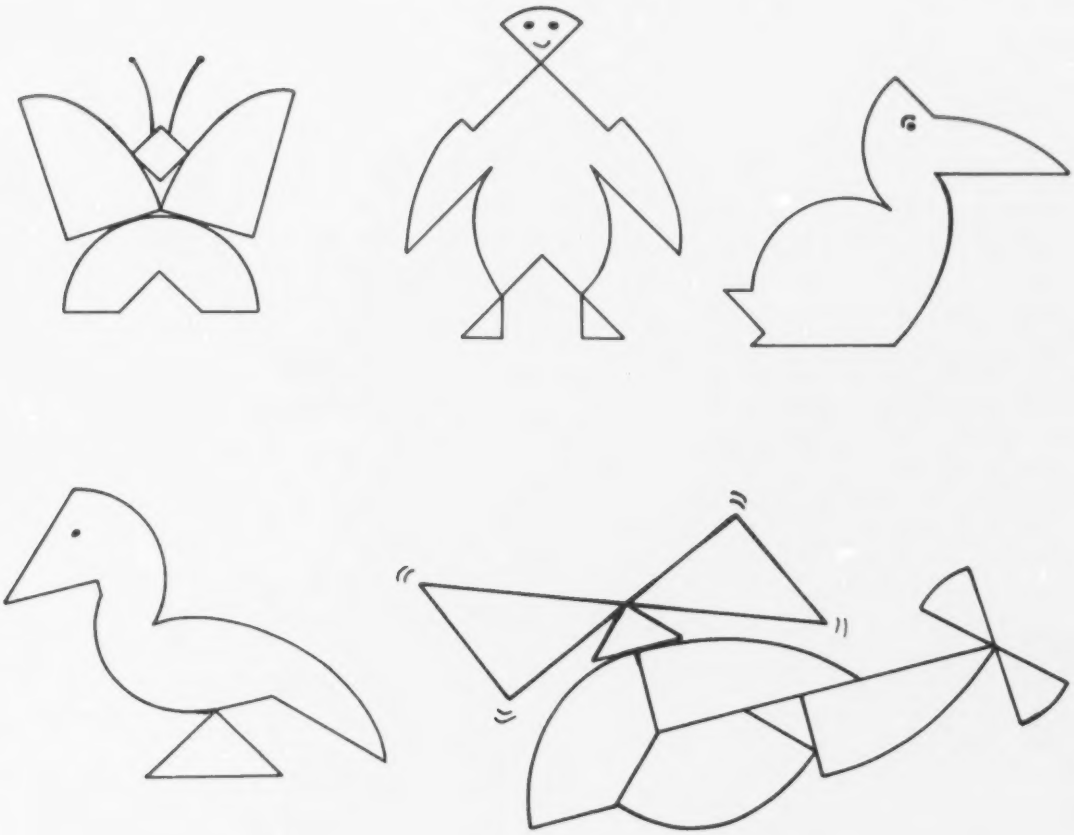
b)
```
   A L L
 H E N S
   L A Y
 -------
 E G G S
 -------
```

c)
```
     D O
   A L L
 H E N S
   L A Y
 -------
 E G G S
 -------
```

12. THE MAGIC EGG



Crack the egg and use the 10 pieces to make the shapes on the next page.



* * *

"I think □ x □ - □ + 17 gives a prime number whatever the number you put in the box".

Is this statement true?

□ = 1	1 x 1 - 1 + 17 = 17	prime
□ = 2	2 x 2 - 2 + 17 = 19	prime
□ = 3	3 x 3 - 3 + 17 = 23	prime

*

SOME DILEMMAS

1. A teacher must cope with about 30 children.	- The majority of useful exchanges occur between a teacher and no more than four children.
2. In order to operate we need a degree of uniformity.	- We want to encourage diversity of activity/response/experience.
3. We recognise that concrete materials benefit most young children.	- We want to encourage children to think about abstractions.
4. We want children to discover things for themselves.	- We want them to come to *some* common conclusions.
5. We want to know what we are trying to do.	- We want to encourage exploration of new situations.
6. We learn from our experience.	- This can lead to stereotyped actions.
7. We want to give children the security which comes from repetition.	- We want children to be able to deal with insecurity.
8. We need to make some plans for children in advance.	- We don't know our children until we have met them.
9. No textbook is satisfactory.	- We cannot make up everything from our own resources.
10. We cannot tell in advance the effect of anything we ask children to do.	- We have to act on the assumption that we can.
11. No teacher wants a syllabus.	- Every teacher wants a syllabus.
12. Mathematics gains its present place in school through its usefulness to society.	- Mathematics earns its place in children's experience through its enrichment of that experience.
13. We want children to understand what they are doing.	- We do many things fluently precisely by *not* thinking about how they work.
14. The range of mathematical activities in which children can now engage is infinite.	- Children have only a finite amount of time in school.
15. We want to know the best way of doing things.	- Often there is no best way.

16. We aim to build up confidence in children.
17. Children should be treated as individuals.

- We are lacking confidence in ourselves.
- Children need the stimulus of other people for their intellectual growth.

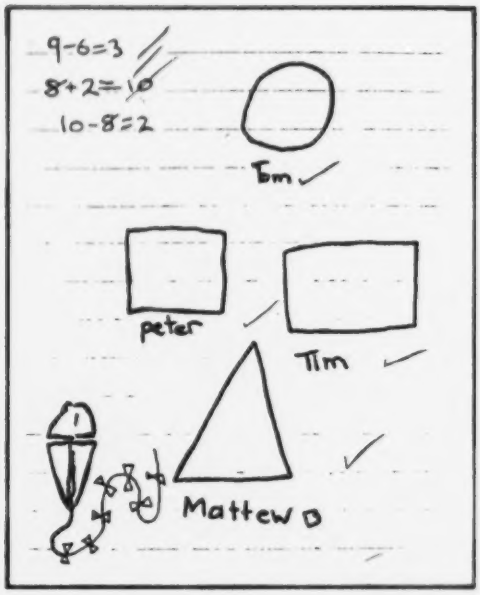
* * *

A WORKCARD

Draw a
circle rectangle
square triangle
Name them.

This is what the teacher set.

This is what a child did!



*

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

ACTIVITIES USING THE TABLES SQUARE

Start at the top left-hand corner of the square. With your finger, trace around the outline of a rectangle.
How many squares in your rectangle? (area)
Can you find the answer written in your rectangle?
Try again with other rectangles.
What did you discover?

* * * * *

Which number appears most often? Why?
Which numbers don't appear at all? Why?
Which numbers appear an odd number of times? Why?
Which numbers appear only once? Why?

* * * * *

Shade all the multiples of 2 on the square.
Shade all the multiples of 3, 4 or 5 etc. on other squares.
Predict what the pattern will be.

* * * * *

Choose any rectangle on the grid.
Multiply together the numbers which are in diagonally opposite corners.
Try for other rectangles.
What did you discover?

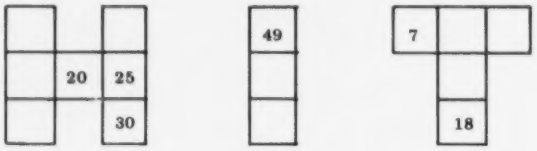
4	5	6
8	10	12

4 x 12 =

6 x 8 =

* * * * *

Fill in the blanks in these extracts from the grid.

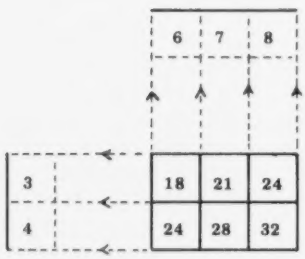


Did you look at the tables square?

* * * * *

a) Select any rectangular block of numbers on the grid.

b) Add the numbers at the beginning of the columns and rows in which the block of numbers appears:
$6 + 7 + 8 = 21, \quad 3 + 4 = 7.$



c) Find the product of these sums:
$21 \times 7 = 147.$

d) Find the sum of the numbers in the rectangle:
$18 + 21 + 24 + 24 + 28 + 32 = 147.$

e) Experiment with rectangles of different sizes.

f) Find the sum of all the numbers on the grid.
$(1 + 2 + 3 + \ldots + 12) \times (1 + 2 + 3 + \ldots + 12) = 78 \times 78$
$= 6084.$

* * * * *

Cut up the square into horizontal bands of numbers.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36

Place the first two bands together and a set of fractions equivalent to $\frac{1}{2}$ is formed.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24

Now take the 3rd and 7th bands and place together

3	6	9	12	15	18	21	24	27	30	33	36
7	14	21	28	35	42	49	56	63	70	77	84

and lo and behold a set of fractions equivalent to $\frac{3}{7}$!

All sets of equivalent fractions up to and including those with a denominator of 12 can be formed.

Suppose we wanted to find the sum of $\frac{2}{5}$ and $\frac{1}{4}$.

Form the set of fractions equivalent to $\frac{2}{5}$ and place beneath it the set of fractions equivalent to $\frac{1}{4}$.

2	4	6	8	10	12	14	16	18	20	22	24
5	10	15	20	25	30	35	40	45	50	55	60

1	2	3	4	5	6	7	8	9	10	11	12
4	8	12	16	20	24	28	32	36	40	44	48

Search through the multiples of 5 and 4 to find a common denominator and then adjust the number bands so that the common denominators are in a vertical line.

2	4	6	8	10	12	14	16	18	20	22	24
5	10	15	20	25	30	35	40	45	50	55	60

1	2	3	4	5	6	7	8	9	10	11	12
4	8	12	16	20	24	28	32	36	40	44	48

$\frac{2}{5} = \frac{8}{20}$ $\frac{1}{4} = \frac{5}{20}$ $\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{13}{20}$

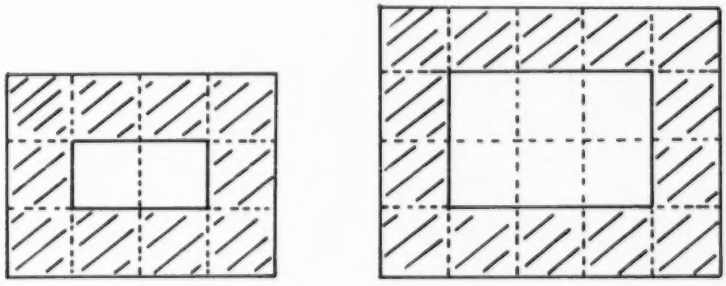
The bands can be used by children as an aid to establish and reinforce the number patterns associated with equivalent fractions. They should be introduced after a great amount of time had been spent modelling fractions using Unifix and other concrete materials, but before the formal algorithms had been established.

It is important to note that the bands can be used to get 'correct answers' without any real understanding of the mathematics involved. However, if children need to manipulate fractions, then the bands will guarantee success and maybe for some children understanding will come later.

* * *

Black and white tiles are arranged to form rectangles in such a way that the black tiles form a border which completely surrounds the white tiles in the centre.

E.g.



a) What is the <u>smallest</u> number of black tiles needed for a rectangle which has 12 white tiles?

b) What is the <u>greatest</u> number of white tiles needed for a rectangle which has 50 black tiles?

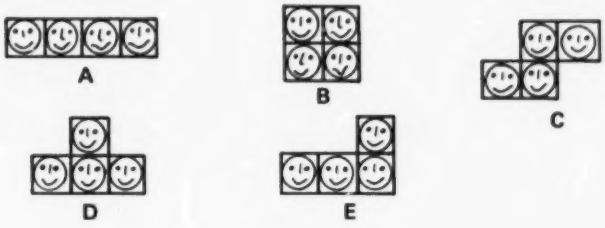
c) Find the length and breadth of a rectangle which uses the same number of black and white tiles?

*

IN HOW MANY DIFFERENT WAYS
CAN YOU BUY 4
ATTACHED STAMPS ?

Children will require cardboard or plastic squares and some squared paper for recording the answers. (This assumes that the stamps are square. Some children may wish to use rectangles.)

Since the stamps can only be joined along a common edge there are 5 basic patterns to consider:



Each pattern can be rotated or reflected and a new arrangement may be found.

E.g.

The number of arrangements for each pattern is: A: 2; B: 1; C: 4; D: 4; E: 8. This gives a total of 19 different ways that you can buy 4 attached stamps.

IN HOW MANY DIFFERENT WAYS
CAN YOU BUY 5
ATTACHED STAMPS ?

* * *

BEING FRIENDLY WITH NUMBERS

There are a number of fundamental rules and ideas in mathematics which are so obvious that their importance and application is often overlooked. A few of these are discussed below.

The Commutative Rule

This states, 'the order in which two whole numbers are added does not affect the result'.

i.e. $6 + 7 = 7 + 6$, $9 + 5 = 5 + 9$ etc.

A similar rule applies for multiplication

$6 \times 7 = 7 \times 6$, $9 \times 5 = 5 \times 9$

The rule obviously does not hold for subtraction and division

$8 - 6 \neq 6 - 8$, $12 \div 6 \neq 6 \div 12$

We, as adults, use the rule without thinking about it

e.g. (i) $6 + 19 =$

In our minds we rearrange it automatically and say $19 + 6 = 25$

e.g. (ii) $25 \times 16 =$

It is easier to think of this as 16 lots of 25.

e.g. (iii) $\frac{2}{3} \times \frac{1}{2}$

Help! What is $\frac{2}{3}$ of $\frac{1}{2}$?

But: what is $\frac{1}{2}$ of $\frac{2}{3}$? obviously $\frac{1}{3}$!

The Associative Rule

The word 'associate' means 'connect' or 'group together'. If we wish to add more than two numbers then it is necessary to group or associate the numbers so that only two are added at any one time.

$6 + 3 + 2$ may be grouped as $(6 + 3) + 2$
or $6 + (3 + 2)$

In each case the operation of addition is performed twice.

The rule states that it does not matter in which way we group numbers for addition, the sum will be the same.

The rule also applies to the multiplication of whole numbers, but not to subtraction and division.

i.e. $2 \times 3 \times 4 = (2 \times 3) \times 4 = 2 \times (3 \times 4)$

but $(6 - 4) - 1 \neq 6 - (4 - 1)$ and

$(16 \div 8) \div 2 \neq 16 \div (8 \div 2)$

E.g. (i) $128 + 19 + 11 =$

This may be grouped as $(128 + 19) + 11$
or as $128 + (19 + 11)$

The second grouping results in an easier computation than the first.

E.g. (ii) $70 \times 60 =$

This is $7 \times 10 \times 6 \times 10$ and three groupings are possible:

$(7 \times 10) \times 6 \times 10$
$7 \times (10 \times 6) \times 10$
$7 \times 10 \times (6 \times 10)$

all of which give the same answer.

Applying the Commutative Rule it can become

$7 \times 6 \times 10 \times 10$

and by choosing $(7 \times 6) \times 10 \times 10$ the calculation is easy.

The Distributive Rule

This rule links the operation of multiplication and addition. It also emphasises place value - the scheme by which numbers are recorded.

E.g. If we can say $3 \times 28 = 84$

then we know $28 = 20 + 8$

and $3 \times (20 + 8) = 3 \times 20 + 3 \times 8$

$$\begin{array}{r} 28 \\ \times\ 3 \\ \hline 84 \\ \hline 2 \end{array}$$

Algebraically, the rule is expressed as

$$a \times (b + c) = a \times b + a \times c$$

We need to be able to use the rule both ways.

E.g.

3 lots of 6 and 4 lots of 6 = 7 lots of 6

4 lots of 7 and 5 lots of 7 = 9 lots of 7

and

8 lots of 16 = 8 lots of 10 and 8 lots of 6

5 lots of 18 = 5 lots of 10 and 5 lots of 8

also

17 lots of 23 = 10 lots of 23 and 7 lots of 23

$$= \begin{pmatrix} 10 \text{ lots of } 20 \\ 10 \text{ lots of } 3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 7 \text{ lots of } 20 \\ 7 \text{ lots of } 3 \end{pmatrix}$$

Inverse Operations

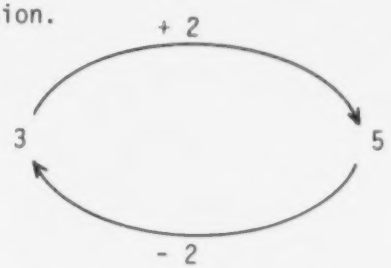
open the door - close the door

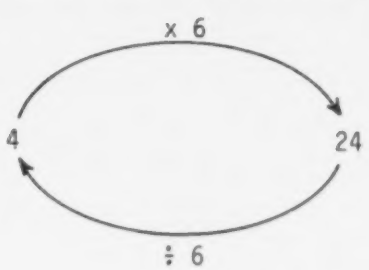
switch on the light - switch off the light

put on your shoes - take off your shoes

These are examples of inverse operations. One action undoes another action.

Addition and subtraction are inverse operations. So too are multiplication and division.





Applying an operation and its inverse to a number, leaves that number unaffected.

7	+ 5 - 5	= 7		12	x 3 ÷ 3	= 12
7	- 5 + 5	= 7		12	÷ 3 x 3	= 12

This idea is often used to make some calculations easier and hence able to be done mentally.

E.g. (i)

8	I say (in my mind).
8	If I add 1 to the 7 it
7	will become 8.
+ 8	4 x 8 = 32
	Because I added 1 - I must now subtract 1.
	Answer is 31.

Notice, between applying the operation +1 and its inverse -1 I did a calculation. This did not affect the outcome but was used to make the calculation easier.

E.g. (ii) 0.3 x 1.2

If I multiply 0.3 by 10 it will become 3.
If I multiply 1.2 by 10 it will become 12
3 x 12 = 36
Because I multiplied by 10 and 10 again
i.e. by 100, I must divide by 100.
Answer = 0.36

E.g. (iii) 2371 - 995

If I take away an extra 5, altogether I
will be taking away 1000.
2371 - 1000 = 1371
Because I took away 5; I must add 5.
Answer = 1376

E.g. (iv) 7 x $1.98

If I add 2¢ to the $1.98 it will become $2.

7 x $2 = $14.

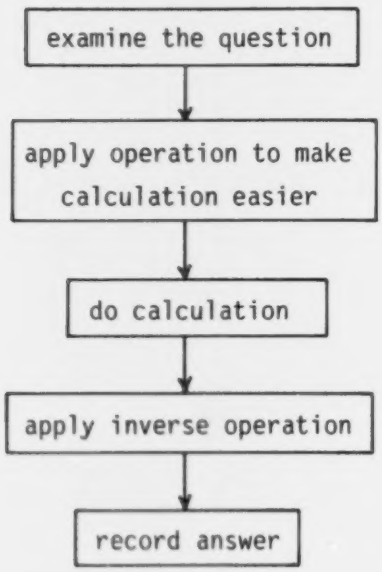
Because I added 2¢ I must subtract 2¢. But I multiplied by 7.

I must subtract 14¢

(7 x 2¢).

Answer = $13.86

The procedure is:



Putting it all together

One of the main aims in mathematics teaching is to get children 'friendly with numbers'. By this, I mean familiar with numbers, rules and operations; knowing how they work; what the possibilities are, etc. It is also the freedom to look at a calculation and be able to make decisions about it

E.g. 112 ÷ 4

Can I do it mentally?

Will I need pencil and paper?

Shall I fetch my calculator?

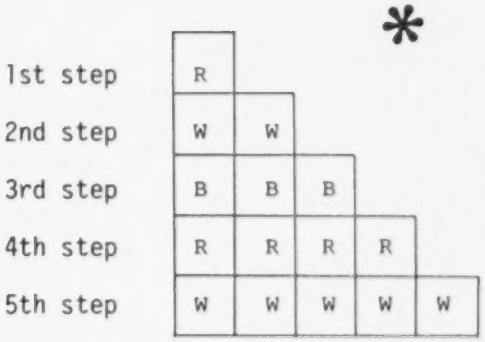
It is obviously not possible to get all children 'friendly with numbers' in the sense described above. However, teachers can:

- encourage children to look carefully at a calculation instead of rushing straight into the formal algorithm;
- encourage children to do some calcuations mentally and then talk about their methods;
- listen, really listen to what children have to say;
- start the day with some mental arithmetic.

Try these: 1 question a day.

248 + 157	\$20 - \$13.34
624 - 389	$\frac{1}{2} + \frac{1}{3}$
15 x 24	8 x \$4.95
680 ÷ 20	75% of \$24

* * *



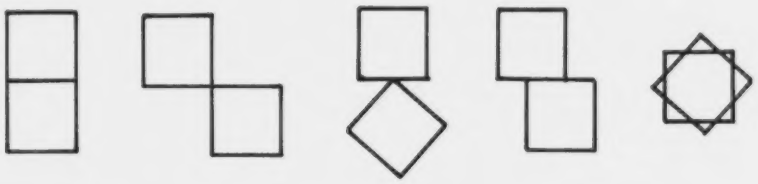
A staircase is made by putting together a large number of red, white and blue cubes.
The 1st step is red, the 2nd step is white and the 3rd step is blue - see the diagram above.
The pattern continues on and on.

a) What colour would the 10th step be?
b) What colour would the 62nd step be?
c) What colour would the 100th step be?
d) If a staircase is made with 30 steps how many red cubes will be needed altogether?

*

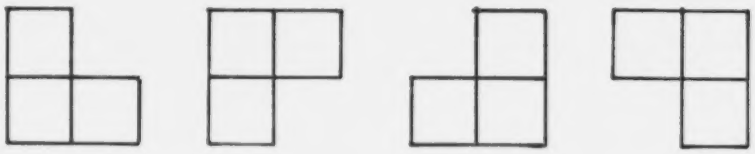
MAKING SHAPES WITH SQUARES

The diagrams below show some ways in which 2 squares can be fitted together.



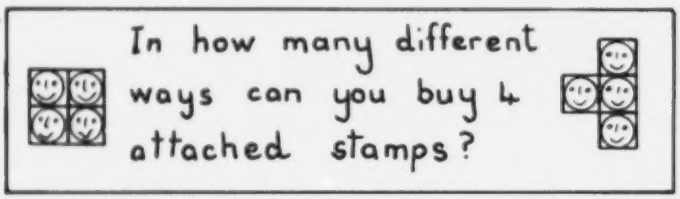
If shapes are made using 3, 4, 5, ... squares it is necessary to introduce some rules in order to limit the number of possibilities. The five work-cards below give examples of activities in which the number of possible shapes has been limited by a particular rule. There will still be a need to make decisions about 'sameness'.

E.g. Are these shapes the same?

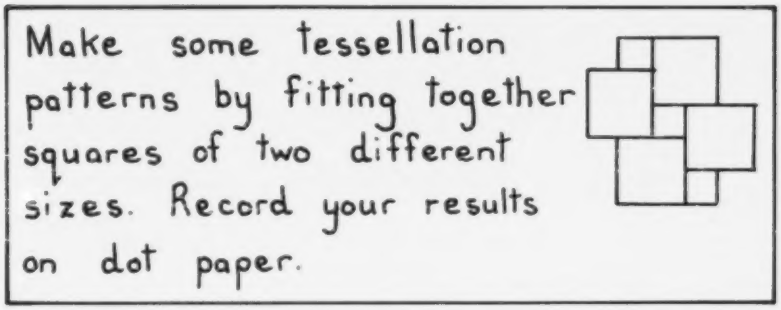


In most cases a reflection or rotation does not produce a new shape. However, see workcard A below and page 115.

A.

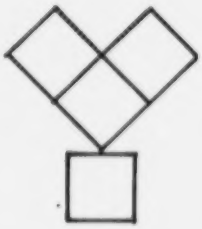


B.



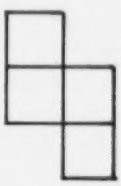
C.

How many different symmetrical shapes can be made by fitting together 4 squares?



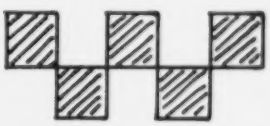
D.

How many different shapes can you cut from squared paper, using only 4 squares?



E.

Look at this picture.
It is made from 5 squares.



The squares are joined only at the corners.

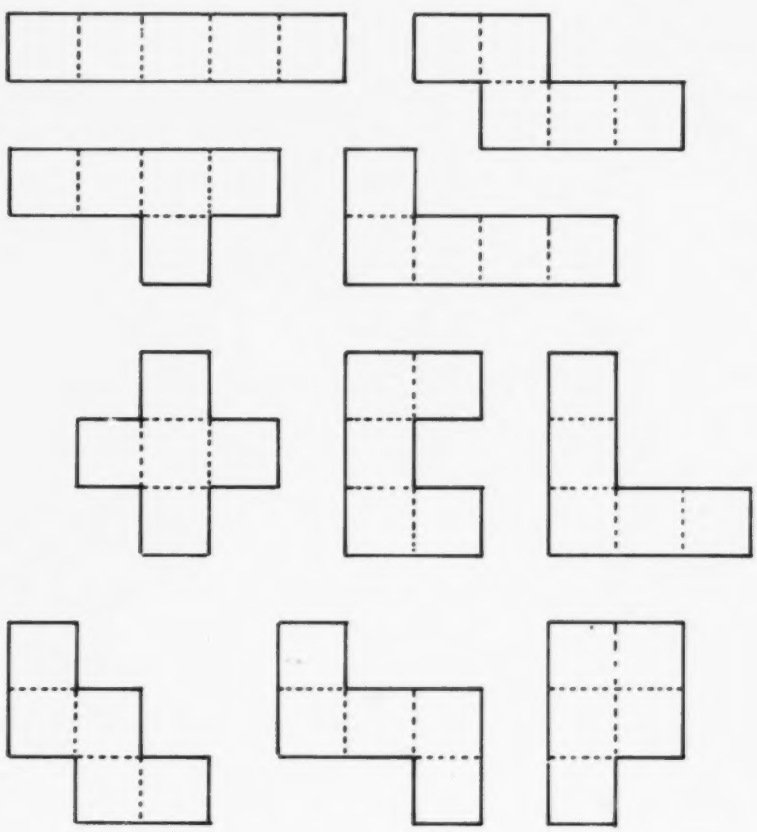
Draw some more pictures of 5 squares joined only at the corners.

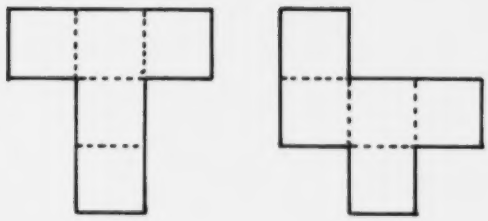
How many different pictures can you draw?

If the squares must be joined exactly along common edges, then the following table gives the name and the number of possible shapes.

NUMBER OF		NAME
SQUARES	SHAPES	
1	1	Monomino
2	1	Domino
3	2	Tromino
4	5	Tetromino
5	12	Pentomino
6	35	Hexomino

The remainder of this article will concern itself with activities using the 12 pentominoes.



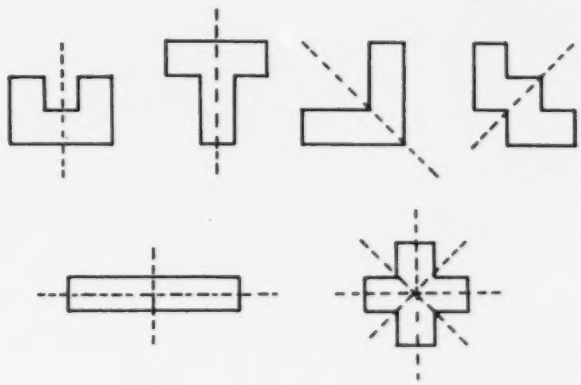


Q. All the shapes have an area of 5 squares. Do they all have the same perimeter?

A. Eleven shapes have a perimeter of 12 units and one a perimeter of 10 units. This is a good activity to investigate with a larger number of squares.
E.g. What are the possible values of the perimeter for a shape made from 8 squares?

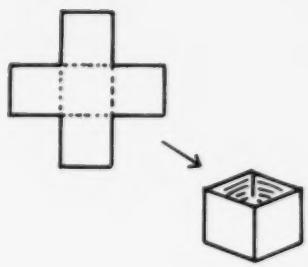
Q. Which shapes are symmetrical?

A.



These can be checked with a mirror.

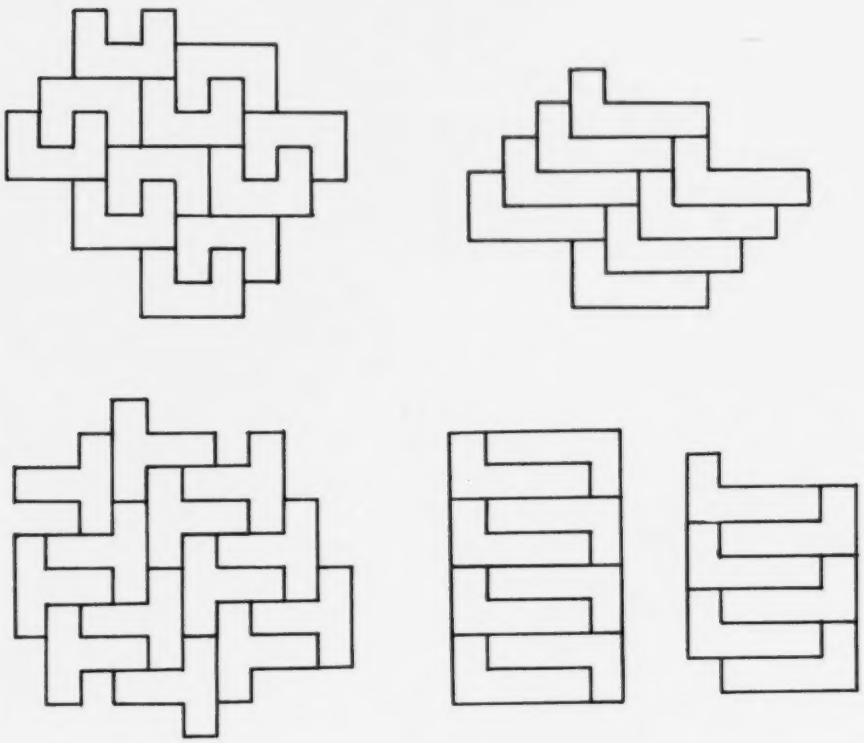
Q. How many of the shapes can be folded to form an open box?



A. Eight of the shapes can be folded into a box.

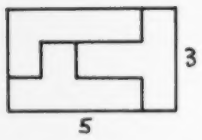
Q. Do all the shapes tessellate?

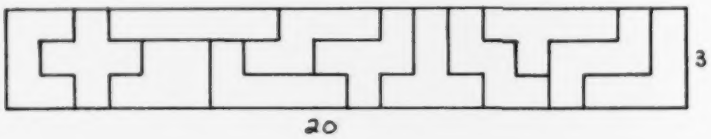
A. Yes. Many of the shapes tessellate in more than one way.



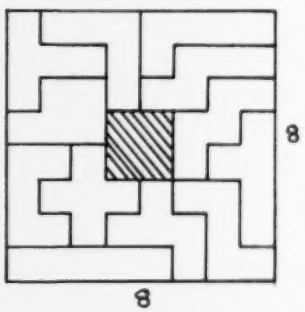
Q. Some of the shapes can be fitted together to make rectangles.
What is the smallest rectangle that can be made?
What is the largest rectangle that can be made?
Is it possible to make a square?

A. The smallest rectangle is 3 x 5 and many different shapes can be combined to produce it. Since there are 12 shapes and each shape has an area of 5 squares, a maximum of 60 squares can be covered with the shapes. 3 x 20, 4 x 15, 5 x 12 and 6 x 10 rectangles can be made using all the pieces.

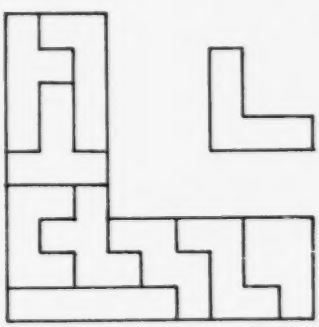


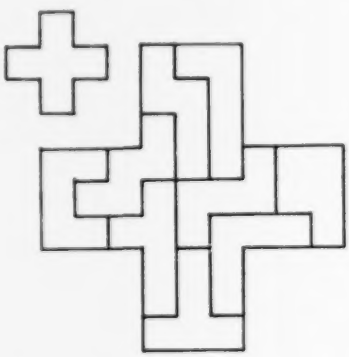


The only square it is possible to make is 5 x 5. An 8 x 8 square with a 2 x 2 'hole' is shown on the right. The square can be made with the 'hole' in different positions.



Q. Choose one of the shapes and draw an enlarged version of it, on squared or dot paper. The length of each edge is to be 3 times that of the original. Using as many of the remaining eleven shapes as necessary, cover the drawing exactly.

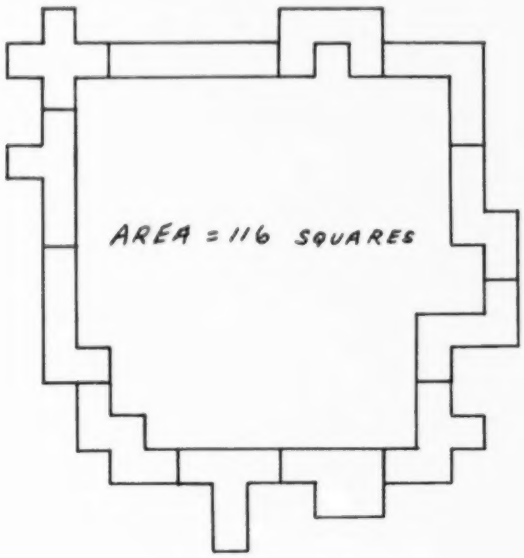




Is it possible to do this activity with all the shapes?

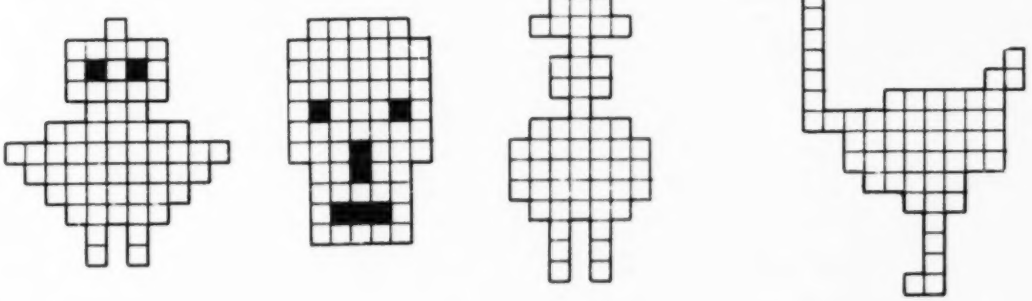
A. Yes. Nine of the remaining shapes will be needed each time.

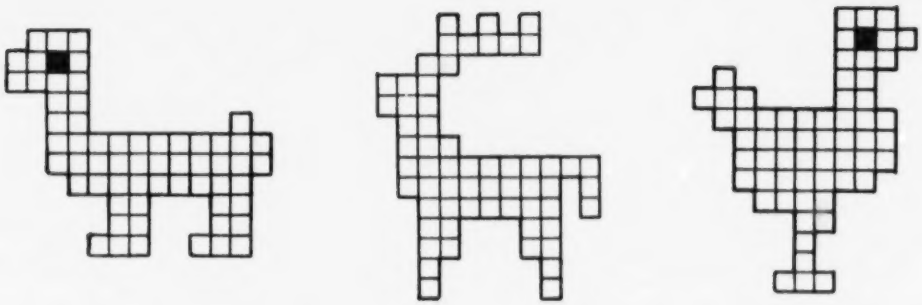
Q. Fit the shapes together (by joining along a common edge) so as to enclose a certain area.
What is the greatest area that can be enclosed?



A. 128 squares. This activity proves to be a real challenge for children and parents.

Use the 12 pentominoes to make each of these shapes:





The Pentomino Game (2 players)

- The game is played on an 8 x 8 square grid. The squares must be the same size as those used for the set of 12 pentominoes.
- Share one set of pentominoes equally between the two players.
- The players take turns placing their pieces on the board.
- The last player to place a piece on the board wins.

Do not teach 'pentominoes'. Rather, allow children to follow a process of guided discovery. It will be necessary for you to ask questions and, through discussion with the children, investigate the possibilities.

Children will need squared or dot paper, cardboard, scissors and, if possible, small plastic squares.

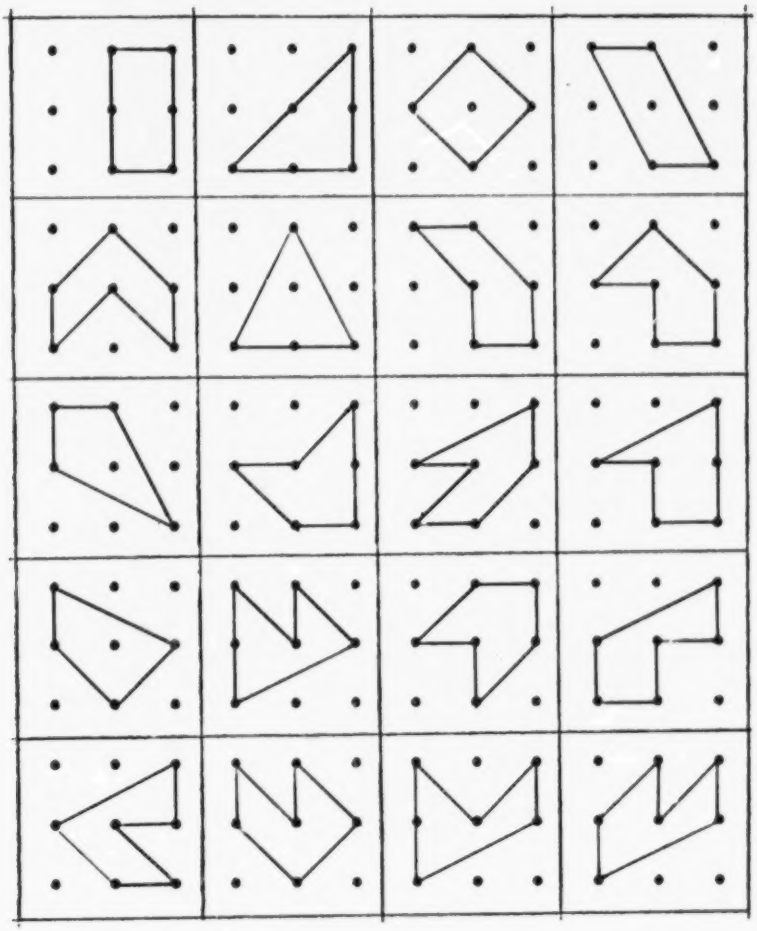
Plastic pentomino sets can be purchased from the games department of K-Mart, Venture and most toy shops. They are sold under various trade names - the most common are:

HEXED - HI-Q PUZZLE by Gabriel
BEAT THE COMPUTER NO. 8 by Tenyo Co. Ltd.

* * *

IN HOW MANY DIFFERENT WAYS CAN YOU FIND HALF THE AREA OF A 3 X 3 NAILBOARD?

Here are a few. Can you find some more?



How many different ways can you divide the 3 x 4 nail-board into 3 equal areas?

* * *

PAL!

PAL is the shortened form of the formula P + A - L =.

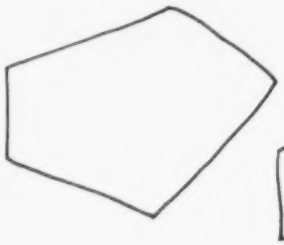
'P' stands for the number of POINTS.
A point is a place where lines meet: or lines intersect:

'A' stands for the number of AREAS.
An area is a piece of surface which is completely enclosed by a number of lines.

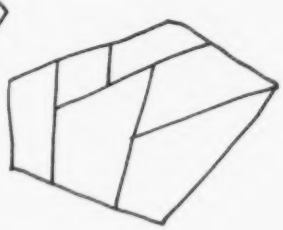
'L' stands for the number of LINES.

A line connects up points.

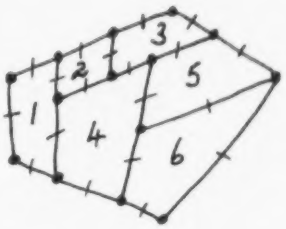
(a) Draw any closed shape.



(b) Divide this large area into smaller areas by drawing lines - as many as you like.



(c) Very carefully count the number of points, areas and lines on the diagram.



P = 14
A = 6
L = 19

(d) Substitute these results into the formula.
P + A - L = 14 + 6 - 19 = 1

No matter how many lines you draw the answer will always be ONE! Try it and see.

* * *